

Basic Kinetic Modeling in Molecular Imaging 2022

Measurement and estimation of flow in brain, heart, liver, muscles, and kidneys using $[O^{15}]H_2O$ PET techniques

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28 Sider

38. Aarg. Nr. 236

30 Øre

Atombombe-Spildstof udnyttes af de danske Læger

To unge københavnske Læger foretager en Række spændende Undersøgelser vedrørende Forkalkning, for højt Blodtryk og Børnelammelse

To unge københavnske Læger modtager med Mellemrum fra England nogle Kolber, som indeholder et Spildprodukt fra Fremstillingen af Atombomber. Det drejer sig om en radioaktiv Luftart, der gaar under Benævnelsen »Krypton 85«, og her i Danmark anvendes Luftarten til en Række læge-videnskabelige Undersøgelser, som man tillægger stor Betydning.

Under Fremstillingen af Atombomber paa det store og hemmelige Fabrikkanlag Harwell i England fremkommer der under Processerne en Del Stoffer, som man ikke kan bruge ved selve Bombe-Produktionen. Derimod har de to unge Læger, den 26-årige Niels Lassen og den 27-årige Ole Munk, fundet en overordentlig nyttig Anvendelse af et af disse Spildstoffer og konstrueret Apparater, i hvilke den nævnte radioaktive Luftart spiller en væsentlig Rolle ved Bestemmelsen af Blodgennemstrømningen i Menneskets Hjerne.

Saadan virker Apparatet, som USA er interesseret i

Da to Læger foretager deres Undersøgelser paa den neurologiske Poliklinik paa Rigshospitalet, hvor B.T. har besøgt dem. Her fortæller de om deres Undersøgelser, hvortil de foreløbig har faaet stillet 25.000 Kr. til Raadighed af Videnskaberne Fond og Chr. X's Fond.

— I Virkeligheden startede det rent tilfældigt, siger de to Læger. Vi blev gjort opmærksom paa, at man var interesseret i at finde frem til en Metode, efter hvilken man hurtigt og sikkert kan bestemme Blodgennemstrømningen i Hjernen.

Vi vidste, at man i Amerika havde en Metode. Men den kunde ikke overføres til danske Forhold. Den amerikanske Metode er analytisk, og den er for langsom og for dyr efter vore Forhold. Vi fandt saa frem til en automatisk Eeregning-metode, som vi nu gennemprøver. Det er vort Haab, at kortlægge, hvor meget Blod der under forskellige Sygdomme strømmer gennem Hjernen. Desuden undersøger vi

meget udbredte, og man ved egentlig ikke ret meget om, hvorledes de opstaar og paavirker Patienterne.

Den rent praktiske Undersøgelse foregaar paa den Maade, at Patienterne indaander almindelig Luft i hvilken vi via forskellige Apparater blander dette Stof »Krypton 85«. Undersøgelsen varer i ti Minutter, og mens den staar paa, tager vi en Række Blodprøver af Patienterne. Vi tager baade Blod fra Arterierne, inden den radioaktive Ilt kommer til Hjernen, og fra Venerne, naar den radioaktive Ilt har passeret Hjernen. Blodprøverne undersøges derpaa i et Apparat, vi har tegnet og ladet fremstille. Det er en automatisk Tællermaskine, hvor man med Geigerrohr kan tælle, hvor meget Blod der er strømmet gennem Hjernen, og se, hvor meget Ilt der er optaget. Vi kan af Gennemstrømningsmængden og den optagne Ilt se, hvor hurtigt Blodet strømmer, og om der undereffs ved Passagen i Hjernen er Hindringer, f. Eks. Forkalkninger.

Med vort Apparat kan der foretages 18 Analyser paa 4½ Time, og dette er meget hurtigere, og vi mener ogsaa sikrere end ved de øvrige kendte Undersøgelsesmetoder. Apparatet har kostet henved 10.000 Kroner at konstruere. Vi fortsætter vore Undersøgelser, og i de sidste Maanedes har vi foretaget Prøver paa flere Patienter. Naar vi har samlet tilstrækkeligt stort Materiale og bearbejdet det, vil vi fremlægge det for Lægevidenskaben, som forhøabeligt saa rent praktisk kan drage Nytte af det. Vi har arbejdet et Aars Tid med den nye Metode, men der vil endnu gaa lang Tid, inden vi er helt færdige.

De to unge københavnske Læger





What is tissue perfusion?

rBF: regional (tissue) blood flow (ml/(100g tissue*min))

rBF is functional, nutritive tissue flow: Blood moving through capillaries

Not rBV: regional blood volume (MR perfusion)

$rBF = rBV / MTT$ MTT: mean transit time (min)



Blood Volume
mL/100g tissue



“Vascular” Blood Flow
mL/min

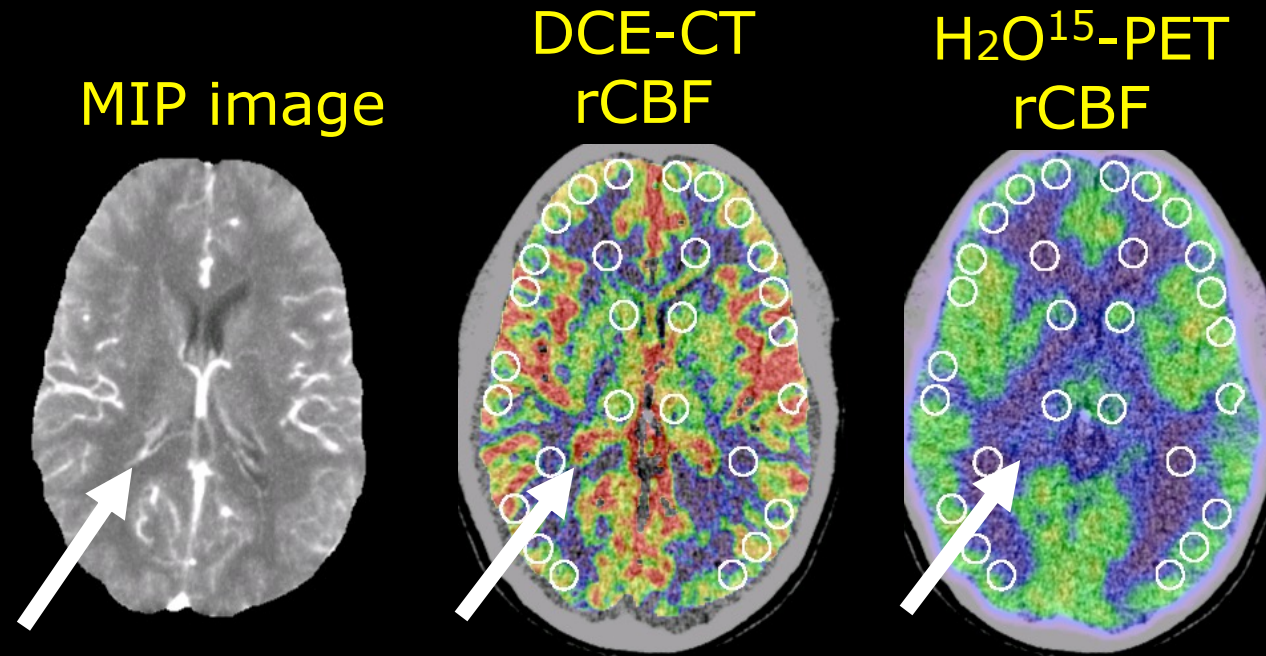


“Tissue” Blood Flow
mL/(100g tissue * min)

Intravascular contrast vs freely diffusible tracer

You can't quantify rCBF with an intravascular tracer

Theory is good for capillary flow, but flow in other vascular structures are messing things up



High flow in vascular structure, but no capillary bed

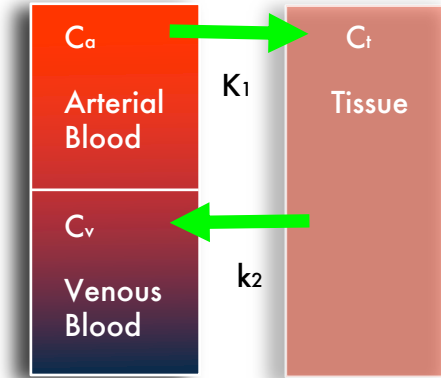


Forces working on the tracer

General considerations: The one-tissue compartment model

Definitions:

p : Partition Coefficient between 2 compartments:
 Ratio of equilibrium tracer concentrations:
 (measured in vitro)



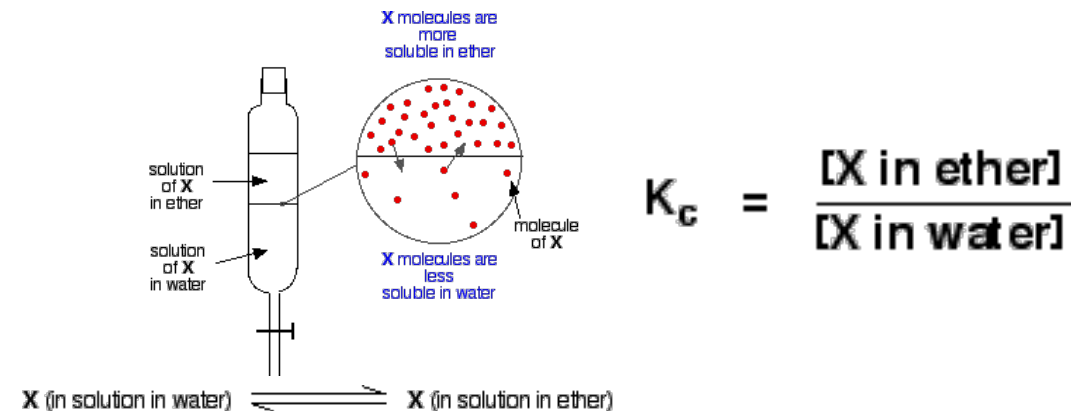
V_t : (Apparent) Volume of Distribution:

Ratio at equilibrium of apparent tracer Volumes to total Volume (Estimated/fitted)

$$K_1/k_2 = C_t(t)/C_v(t) = V_t \approx p$$

[unitless; g/g; ml/g; ml/ml]

At venous side almost diffusion equilibrium





Values for Partition Coefficient

TABLE 1. Partition coefficients for whole brain ($\bar{\lambda}$), gray matter (λ_g), and white matter (λ_w)

	Dimensions for partition coefficient values		
	g/g	ml/g	ml/ml
$\bar{\lambda}$	0.96	0.90	0.95
λ_g	1.04	0.98	1.03
λ_w	0.87	0.82	0.86

TABLE 2. Brain partition coefficient of water as a function of hematocrit

Hematocrit (%)	Water content of blood (g/ml)	λ_g (ml/g)	λ_w (ml/g)	$\bar{\lambda}$ (ml/g)
25	0.895	0.94	0.78	0.86
35	0.873	0.96	0.80	0.88
45	0.851	0.99	0.82	0.90
55	0.829	1.01	0.84	0.93

λ_g , λ_w , and $\bar{\lambda}$, partition coefficients for gray matter, white matter, and whole brain, respectively.

Tissue Hematocrit Pathology

TABLE 3. Regional values of the brain partition coefficient of water

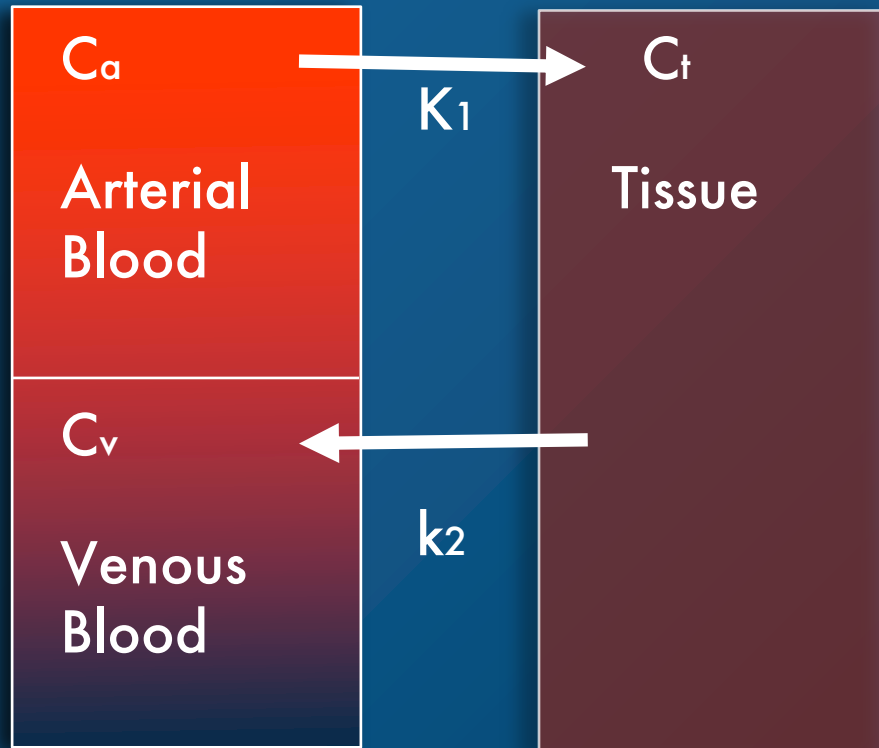
	Water content (g/100 g)	Partition coefficient (ml/g)
Gray matter		
Cerebral cortex	84.3 ^a	0.99
Thalamus	75.1 ^a	0.88
Caudate nucleus	81.4 ^b	0.95
White matter		
Centrum semiovale	70.7 ^a	0.83
Corpus callosum	75.7 ^a	0.89
Edematous centrum semiovale	81.8 ^a	0.96

^a Data from Stewart-Wallace (1939).

^b Data from Randall (1938).

How to measure regional Blood Flow

General considerations: The one-tissue compartment model



A Freely Diffusible Tracer

$$\frac{dC_t(t)}{dt} = K_1 C_a(t) - k_2 C_t(t)$$

Δ Tissue conc = Influx - outwash

Solution:

$$C_t(t) = K_1 C_a(t) \otimes e^{-k_2 t}$$

How many passengers at a give time, t ?

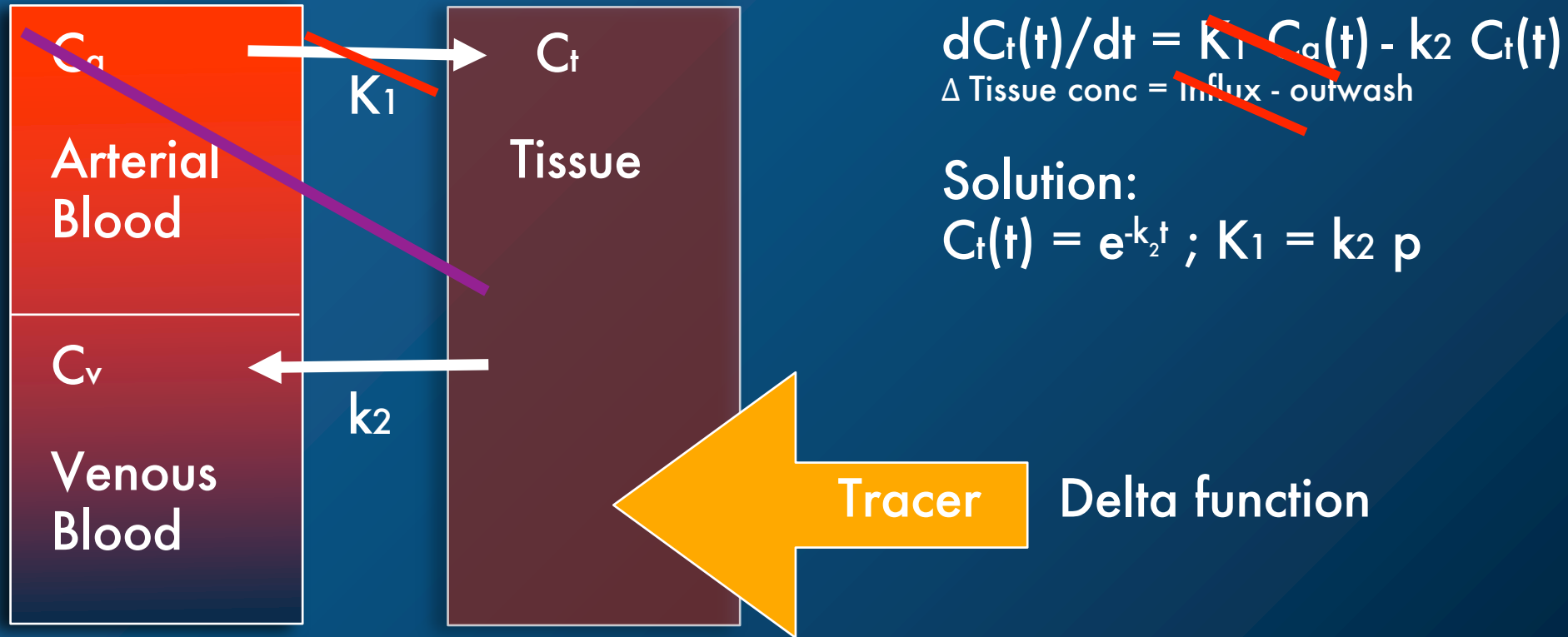


k_2 [min^{-1}]: Washout Rate Constant

K_1 = Unidirectional clearance of blood from tracer, influx rate [$\text{ml g}^{-1} \text{min}^{-1}$].

Will return to this later

Special considerations: only washout



Only Washout/Tissue clearance curve/Desaturation



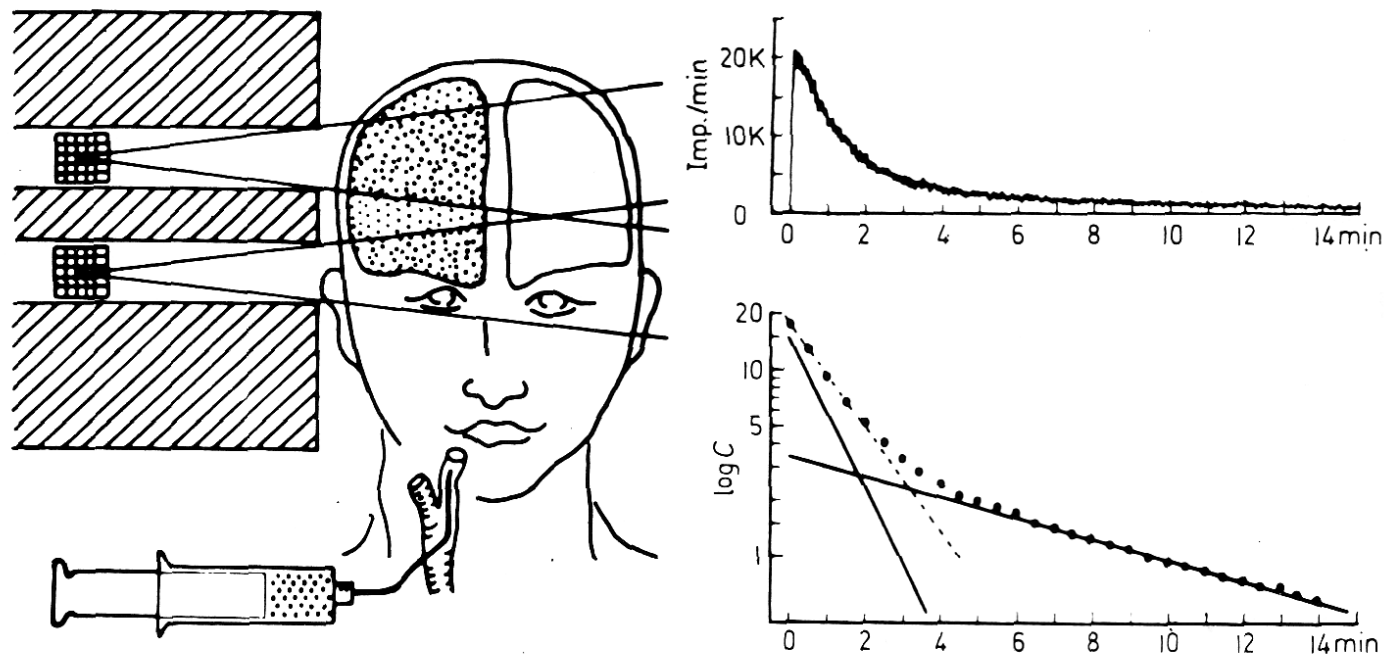
Xenon clearance method

Only Washout/tissue clearance

Residue detection during desaturation

Fraction of bolus in tissue as function of time

$$K_1 = k_2 p = \ln(2)/T_{1/2 p}$$

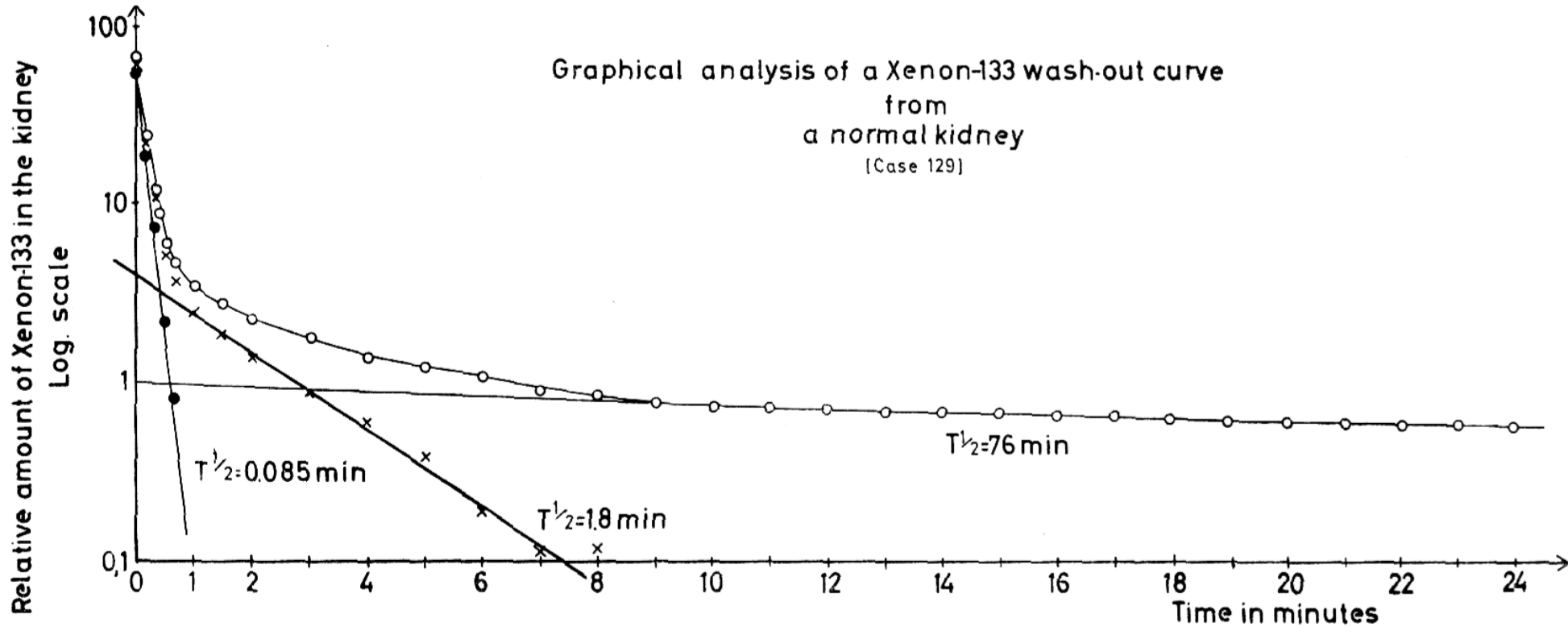


Grey matter: 80 ml/(100g min)
White Matter: 20 ml/(100g min)

Fig. 2. Xenon-133 used for two-dimensional CBF in man: the Lassen-Ingvar intra-arterial injection technique.



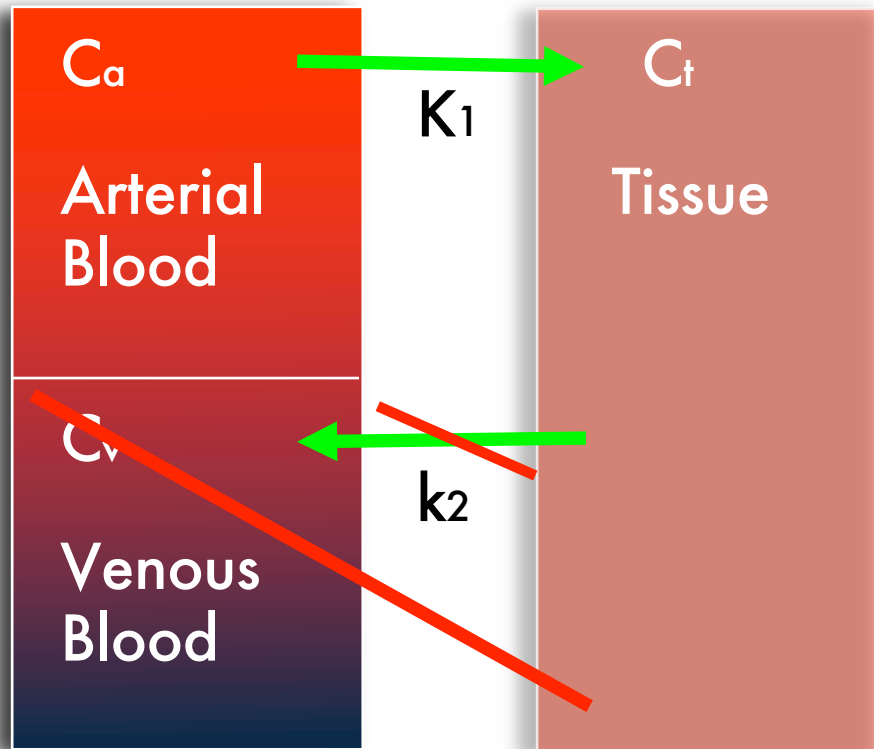
Xenon clearance method



- Skin, Muscles, Fat tissue, Tendons



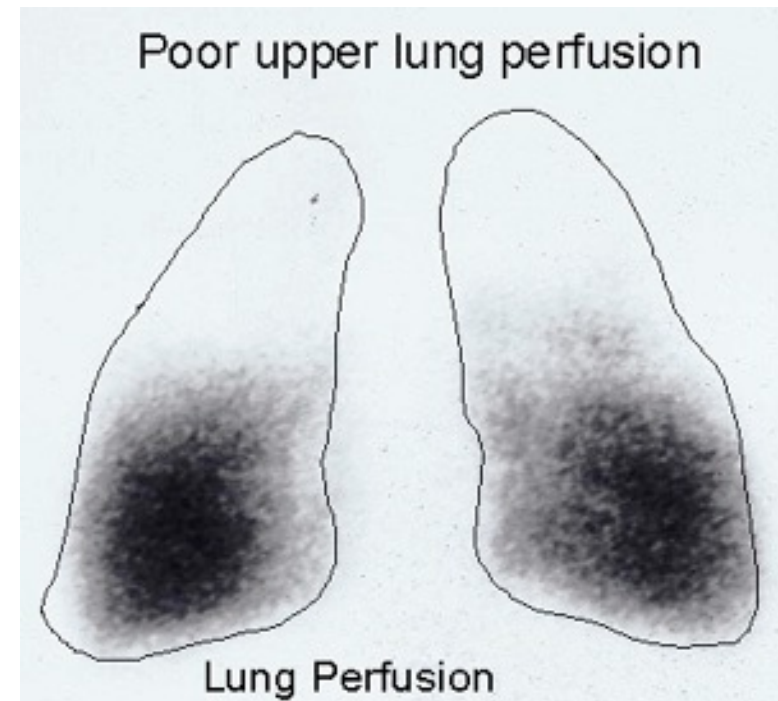
Special considerations: no washout



$$\frac{dC_t(t)}{dt} = K_1 C_a(t) - \cancel{k_2 C_t(t)}$$

Δ Tissue conc = Influx - ~~outwash~~

$$C_t(t) = f C_a(t)$$



Perfect Trapping:
Example: Microspheres



Regional Cerebral Blood Flow in Pigs Estimated by Microspheres

F. F. Madsen¹, F. T. Jensen², M. Væth³, and J. Ch. Djurhuus⁴

- 1) MS = small spheres 15 μ m labelled radioactively or fluorescence
- 2) inject in left ventricle of heart - sample arterial blood in abdominal aorta over time
- 3) Slices are cut out and activity counted

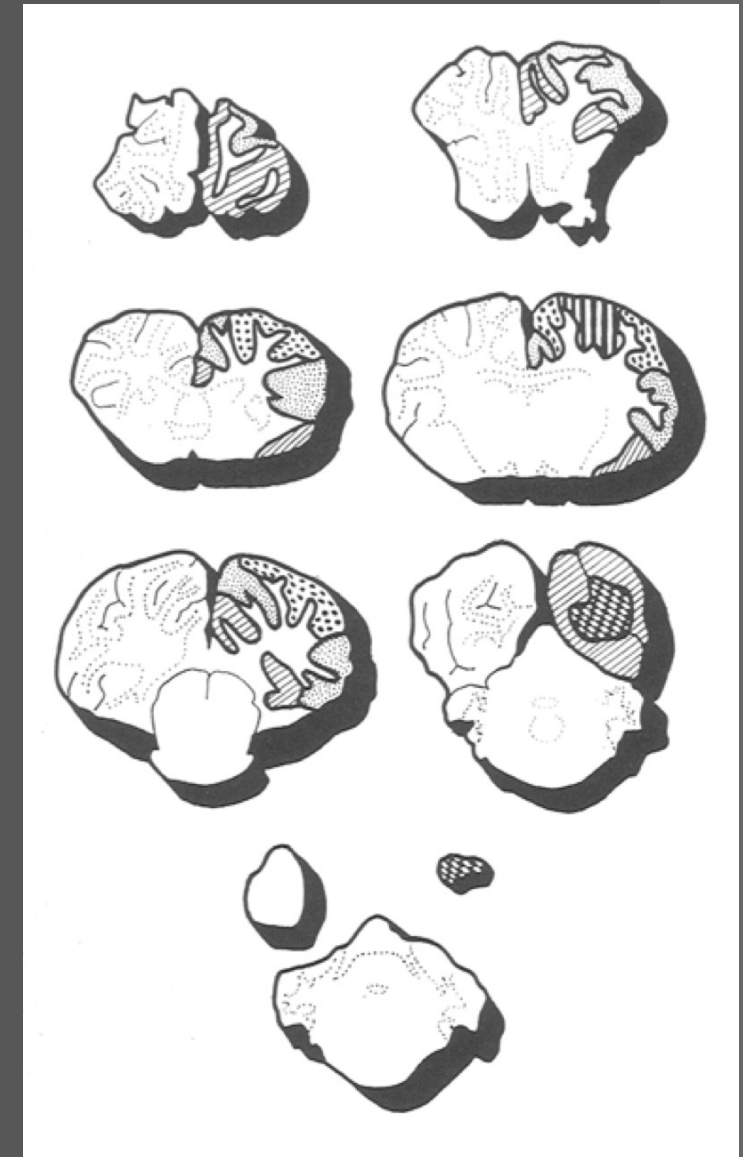
Flow was calculated according to the formula:

$$F_i = N_i \times RF / A_t$$

where F_i is flow in the region of interest ($\text{ml}/\text{min} \times \text{g}$), N_i the activity in the region of interest ($\text{counts}/\text{min} \times \text{g}$), RF the sampled volume in the reference test (ml/min) and A_t the total activity in RF (counts/min).

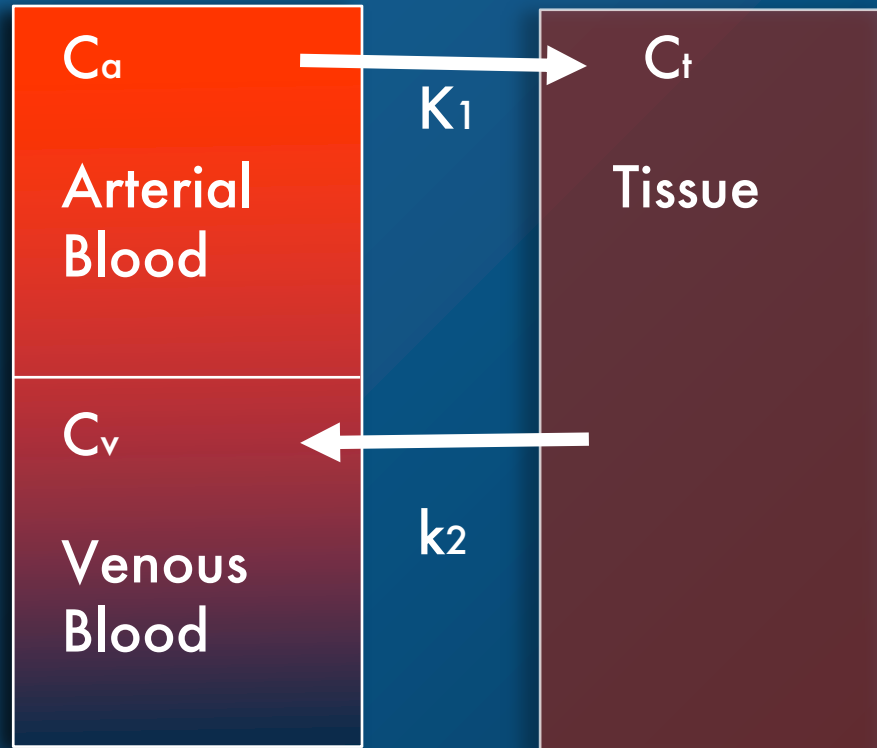
Assumptions:

- 1) an even mixing of the MS in the blood
- 2) the MS should be trapped proportionally to the regional cerebral blood flow
- 3) the MS should be trapped during first passage, (0.5 % i sag sin)
- 4) the MS should not interfere with rCBF or regional physiological steady state.



How to measure regional Blod Flow

General considerations: The one-tissue compartment model



A Freely Diffusible Tracer

$$\frac{dC_t(t)}{dt} = K_1 C_a(t) - k_2 C_t(t)$$

Δ Tissue conc = Influx - outwash

Solution:

$$C_t(t) = K_1 C_a(t) \otimes e^{-k_2 t}$$

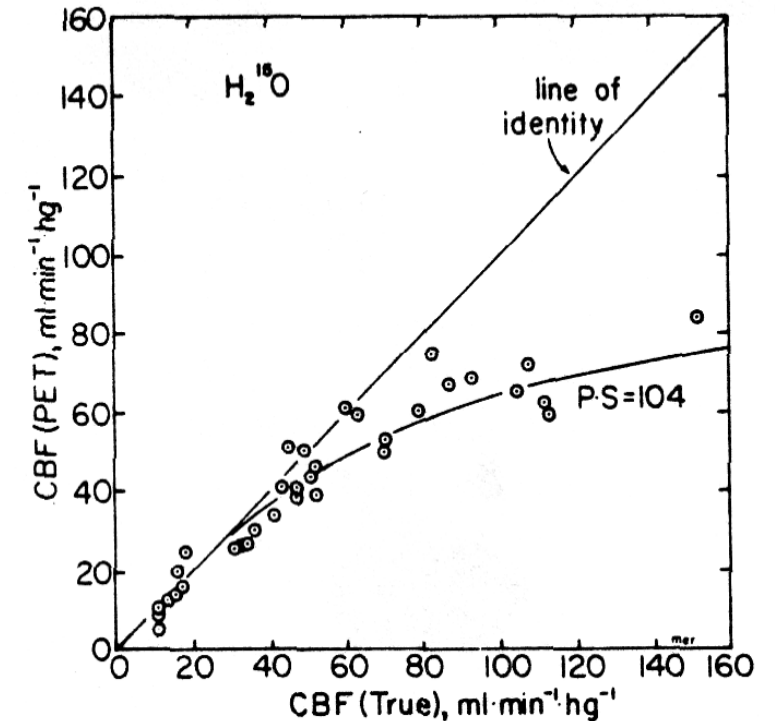
K_1 = Unidirectional clearance of blood from tracer, influx rate [$\text{ml g}^{-1} \text{min}^{-1}$].

k_2 [min^{-1}]: Washout Rate Constant: E: extraction fraction

$K_1 = E f$, if $E=1 \Rightarrow K_1 = f$ (rCBF, Perfusion)

Assumptions - Single Tissue Compartment Model

- Tracer do not influence physiological processes
- Constant physiological state during measurement
- $[^{15}\text{O}]\text{H}_2\text{O}$ is freely diffusible across BBB – **no: $E = 0.87$**
- Compartmental Instantaneous mixing
- ROI filled with 1 tissue with homogenous tracer conc.
 - Free exchange between tissue water og $[^{15}\text{O}]\text{H}_2\text{O}$
 - No other tissues in ROI (Vascular, White Matter, CSF): **no: limited resolution**

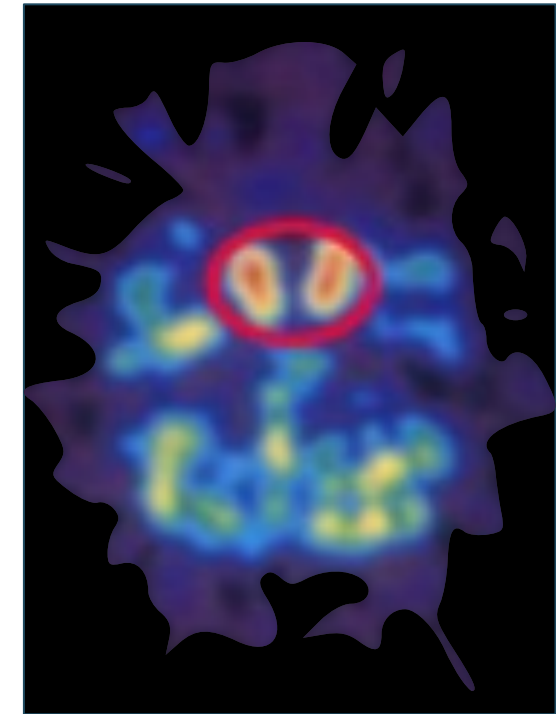
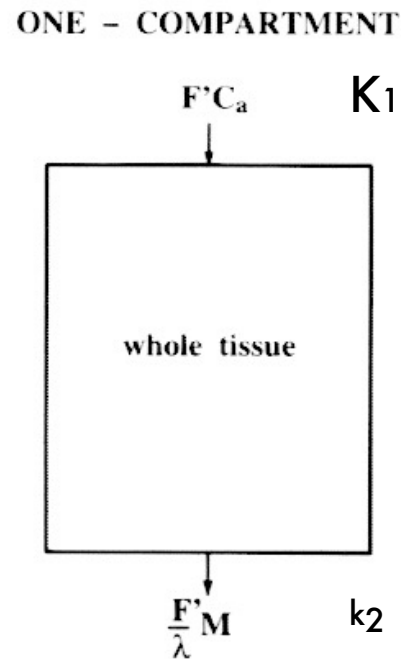
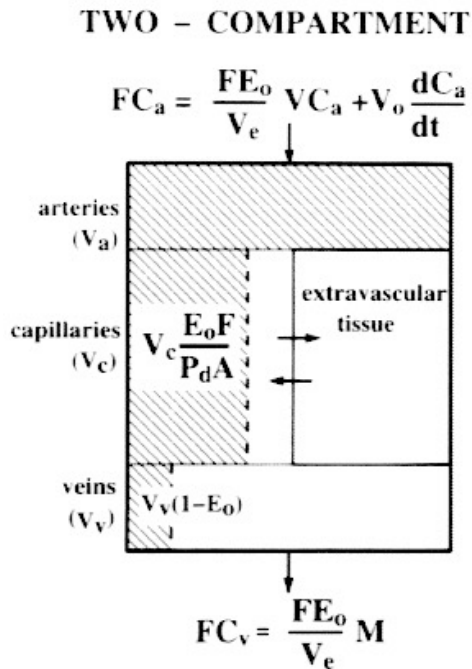




How do we deal with unextracted [¹⁵O]H₂O?

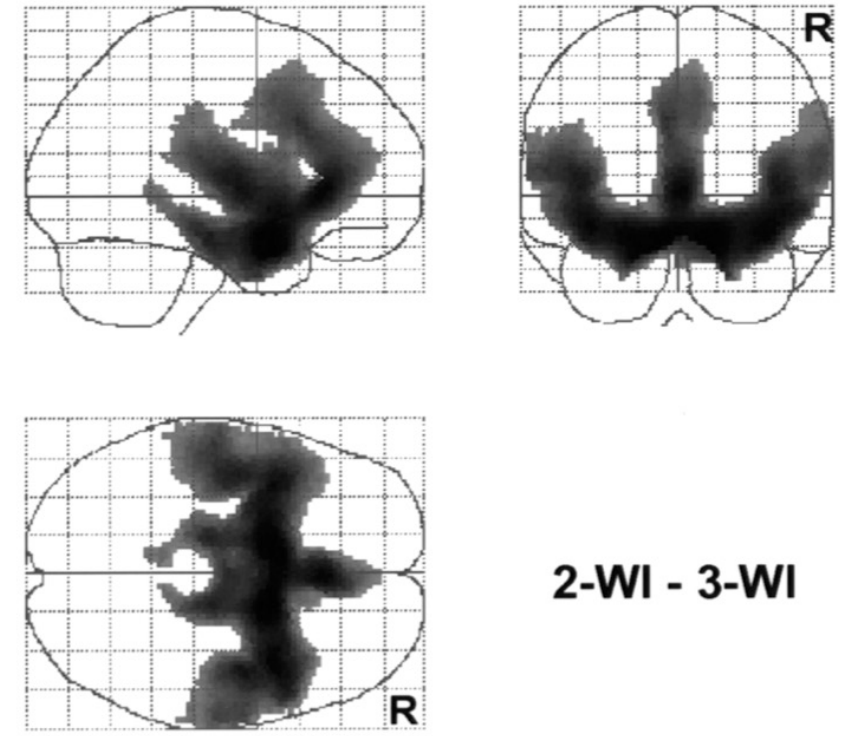
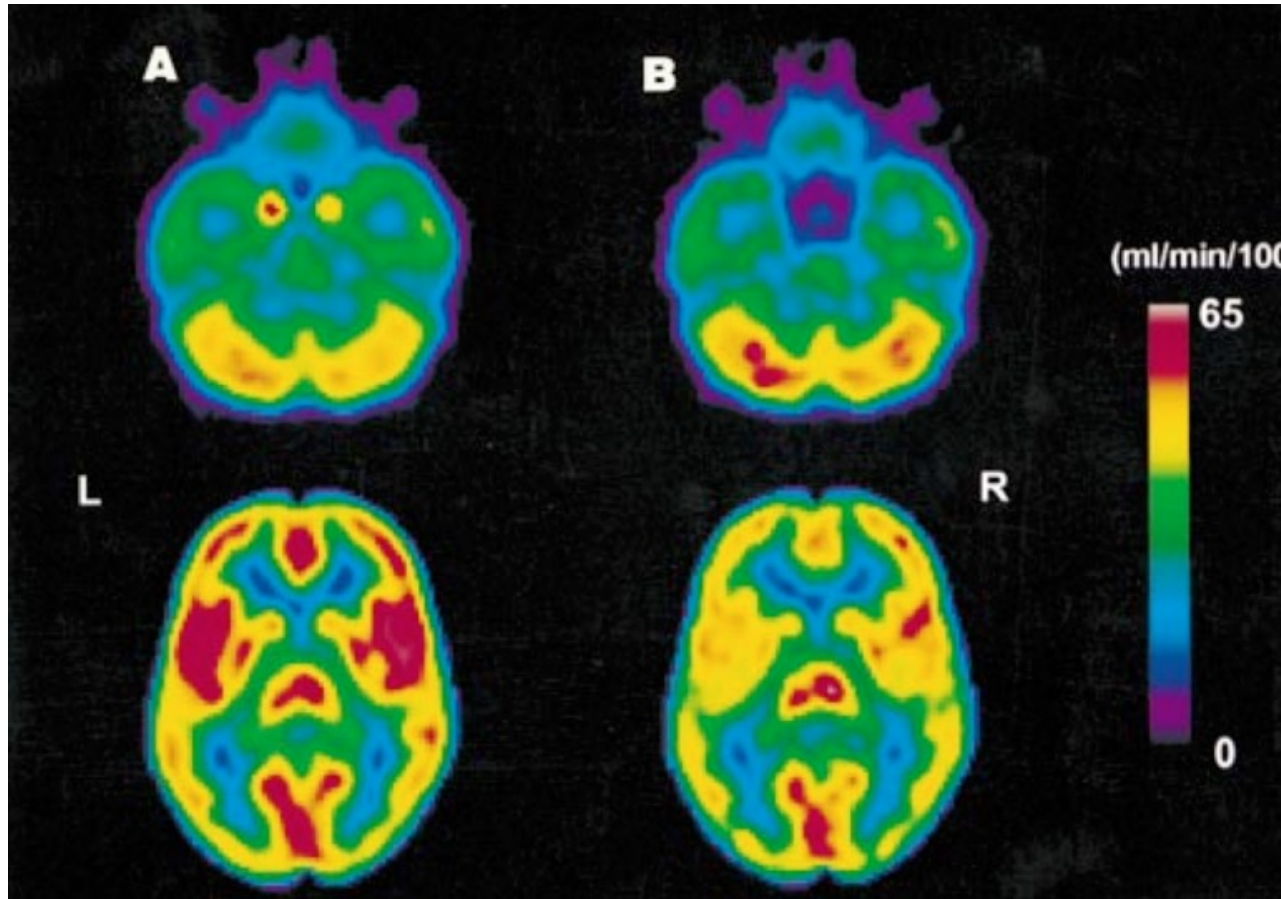
V_b: arterial vascular fraction: correct for "image signal that look like arteries"
 Present standard: 1 tissue 2 compartment model (1T2K_{v_b}), 180 sec acquisition

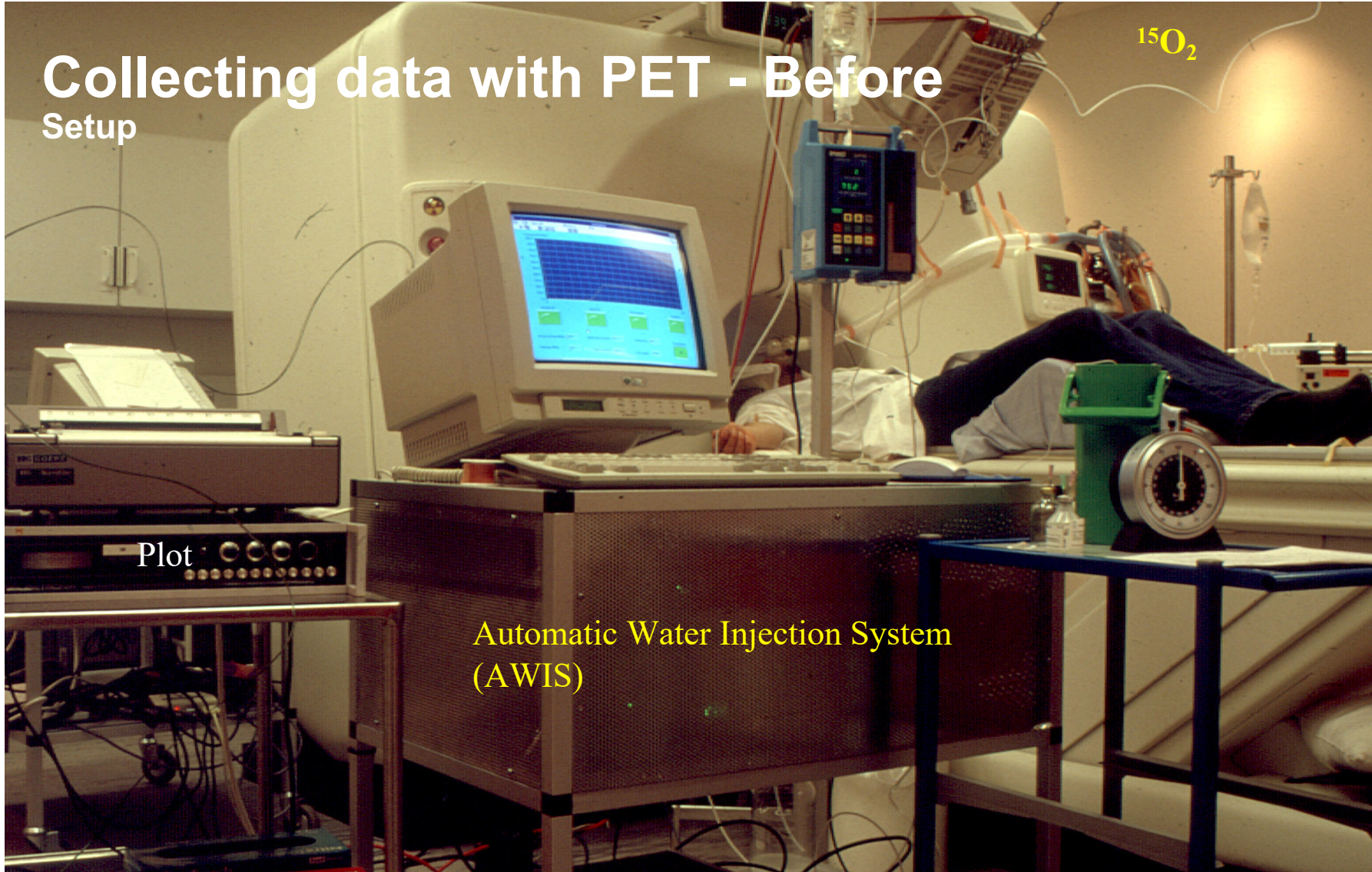
$$C_t(t) = C_a(t) V_b + (1 - V_b) K_1 C_a(t) \otimes e^{-(K_1/V_t)t}$$





Effects of correcting for Vb on rCBF quantification





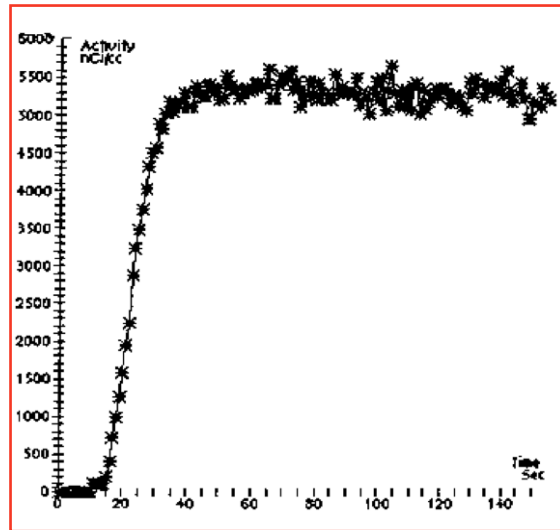
Collecting data with PET - Before Setup

Plot

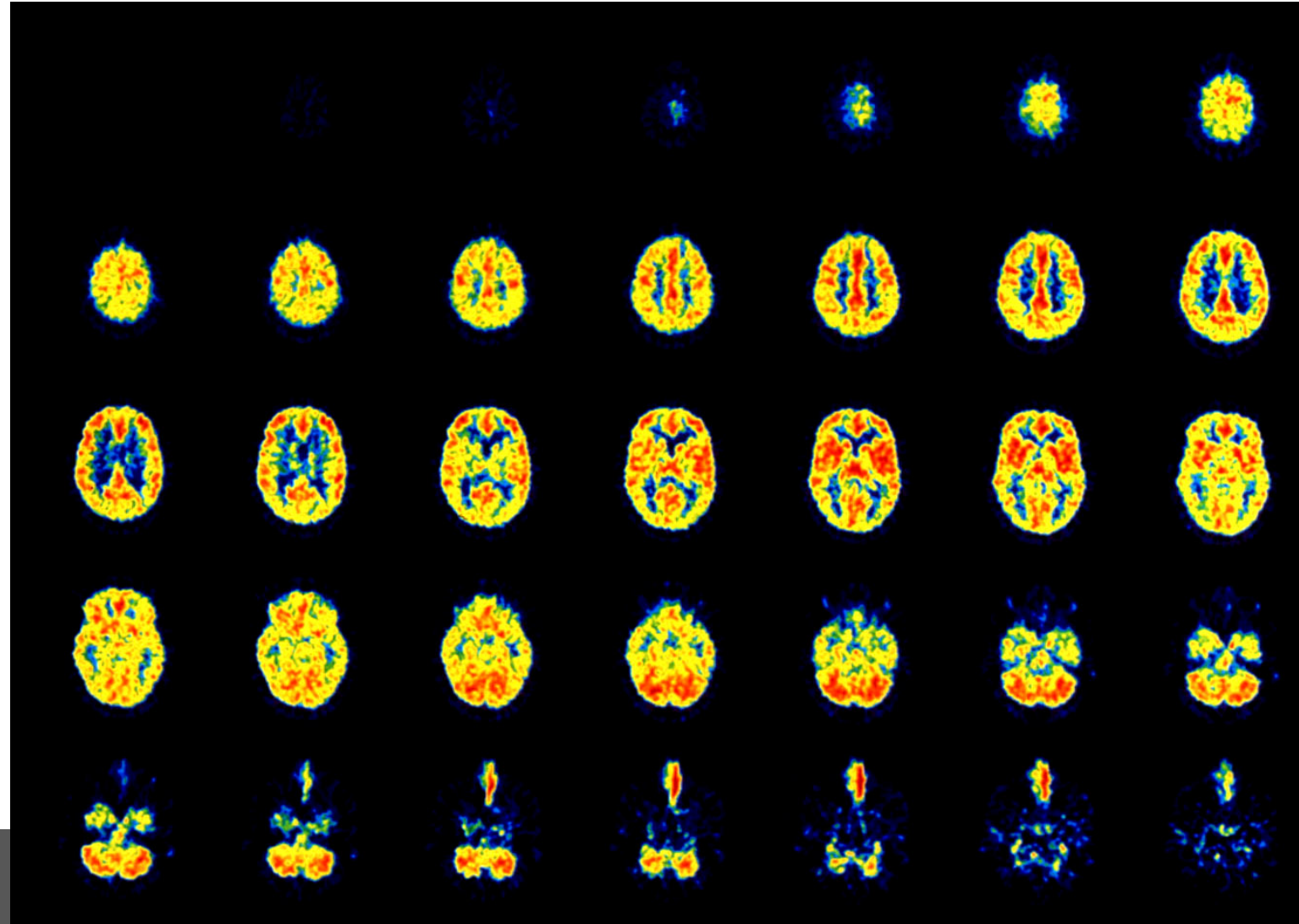
Automatic Water Injection System (AWIS)



Regional Cerebral Blood Flow - 90 sec distribution image, PET [^{15}O]H $_2\text{O}$

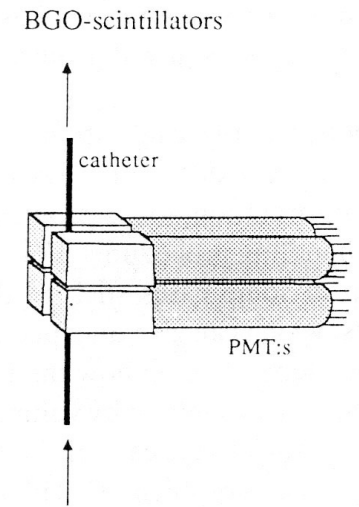
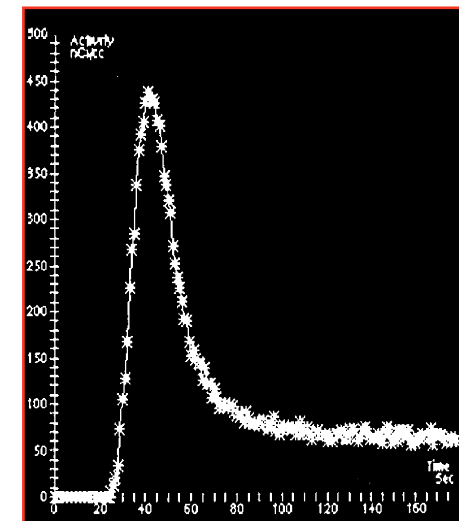
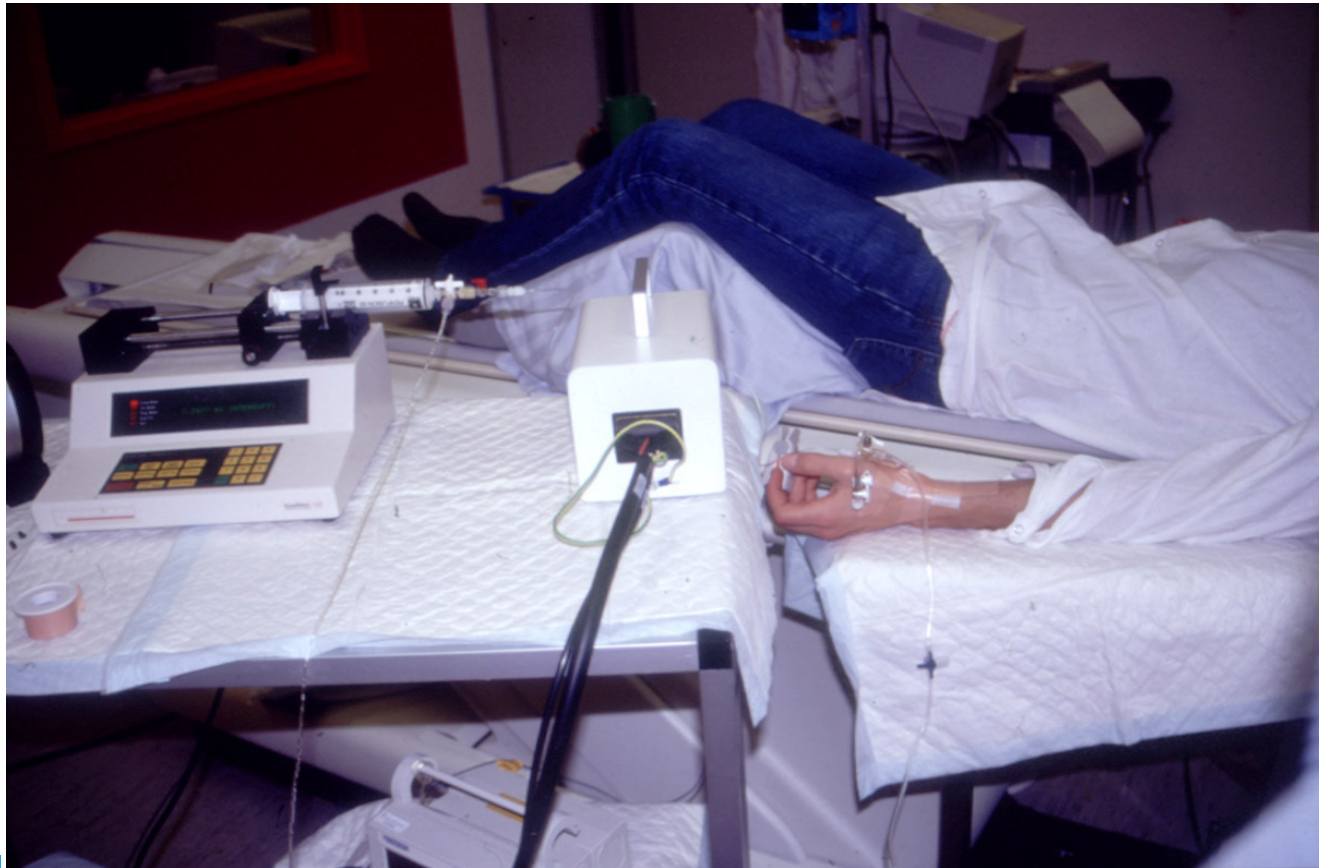


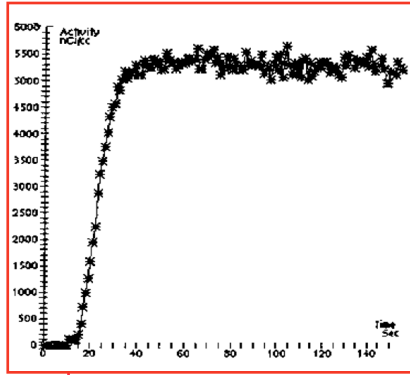
Observed Tissue
Response function $C_t(t)$





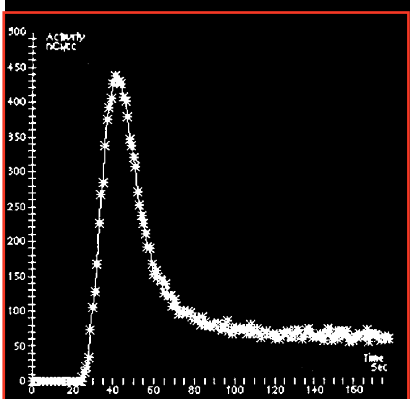
Arterial input function ($C_a(t)$): Applied test function





Input Function- adjustment

Response function from brain tissue, $C_t(t)$

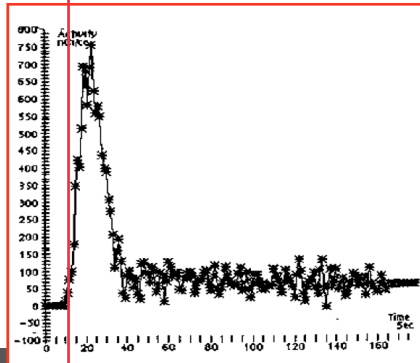


Original input from artery, $C_a(t)$, AIF

Problem?

$$O^{15} T_{1/2} = 2\text{min}$$

rCBF Overestimation $CBF \approx AUC(\text{Tissue})/AUC(\text{AIF})$

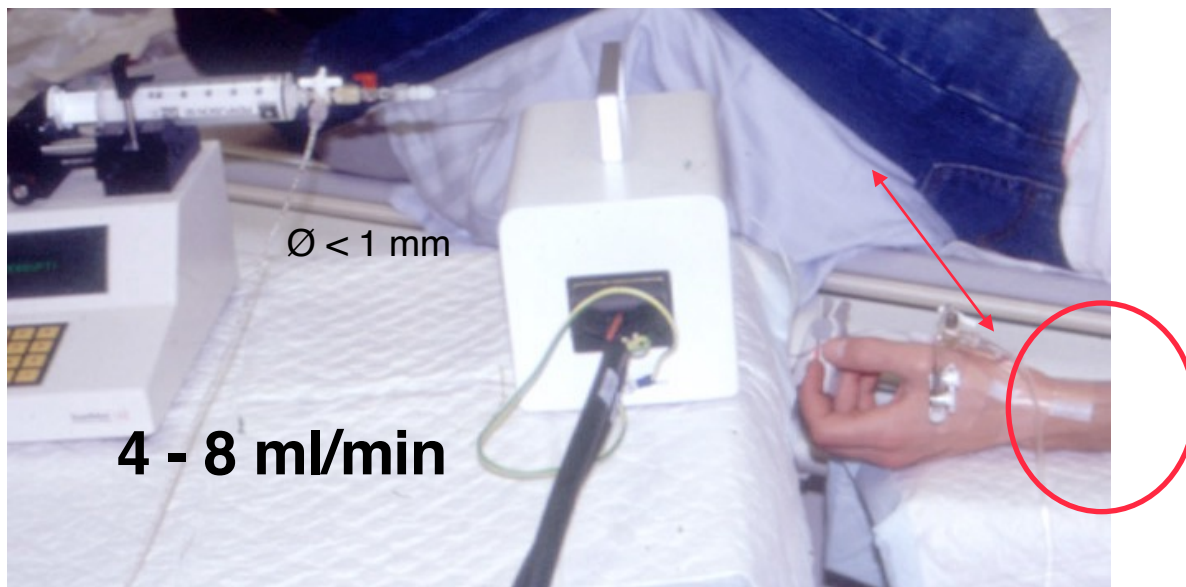


Delay & Dispersion (=Smear) corrected input function

Calculated for whole brain - Assumed to apply regionally



Dispersion Sources = "smear"



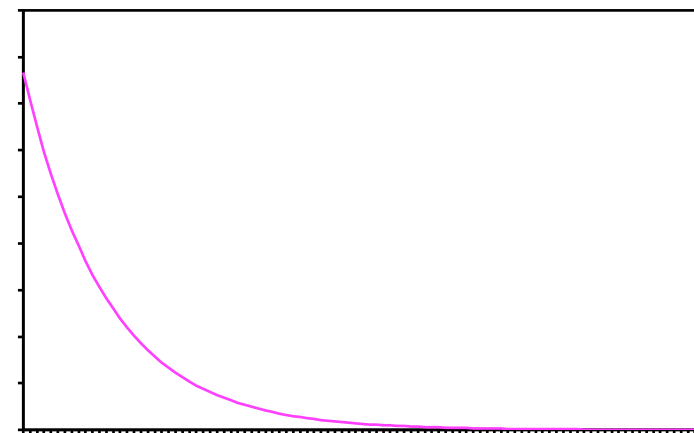
Internal:

- Vascular transport to cannulation

External:

- Cannulation, 3 way valve
- Diameter of tube (< 1 mm)
- Speed of aspiration (4-8 mL/min)

Dispersion function



τ : Dispersion parameter

$$d(t) = \frac{1}{\tau} \exp \frac{-t}{\tau}$$



Arterial cannulation – gold standard, but prefer not to

- Trauma:
 - Invasiveness
 - low, but non-negligible risk of complications
- Robustness:
 - Artery Access
 - Anticoagulant therapy
 - Patient Acceptability
 - Patient Fragility
 - Children
 - Chronic disease
- Cost:
 - Anesthesiologist
 - Logistic/Planning
 - Staff training/availability
 - Physicist/techs
- Technical issues:
 - Calibrating 2 different measurement devices (scanner/sampler)
 - Measurement
 - Clocks
 - Noise induced by corrections



New setup - Using the big scanner - Total body PET/CT



- Siemens PET/CT Quadra – x 4 detector rings



26 cm
106 cm

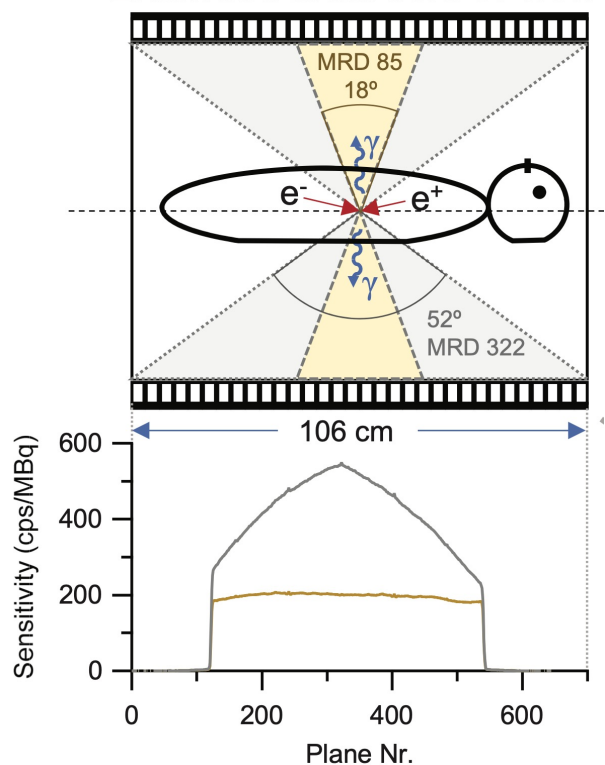


400 MBq [¹⁵O]H₂O 12 min dynamic acq

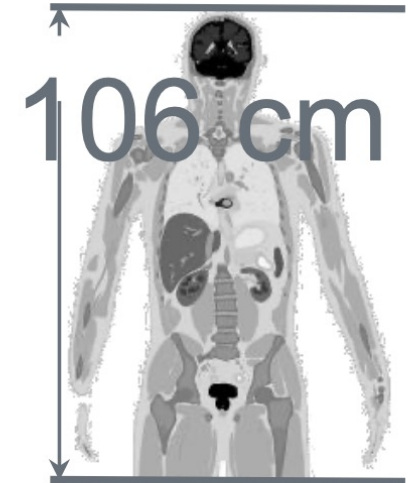


Effects of large axial field of view (AFOV)

Large acceptance angle with
Extended axial field of view



- Increased sensitivity = more counts x (10-30)
 - Increased temporal resolution (1 sec (msec?)) or
 - Better image quality or
 - Decrease in acquisition time or
 - Increase in acquisition time or
 - Lower dose injected
- Increased coverage
 - Image derived input function (IDIF)
 - Simultaneous multiorgan acquisition



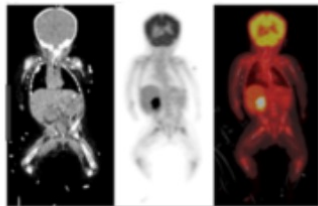
Whole-Body PET/CT Imaging - Perspectives

High-sensitivity



Fast acquisition

Low radiation burden



Quantitative accuracy

Multi-tracer studies

Organ interaction, e.g. brain-gut, brain-spine, brain-heart

Late low-dose Imaging
→ immune cells

Drug discovery

High temporal resolution

Whole-body parametric imaging

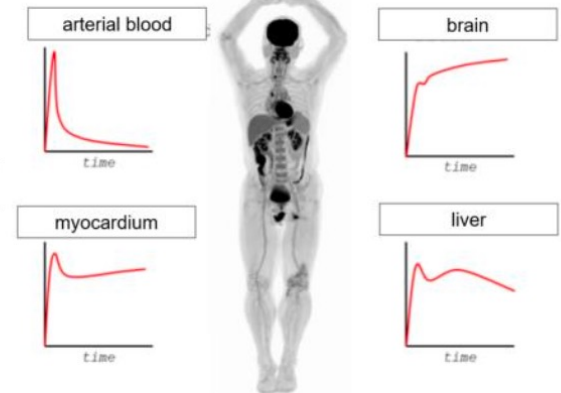
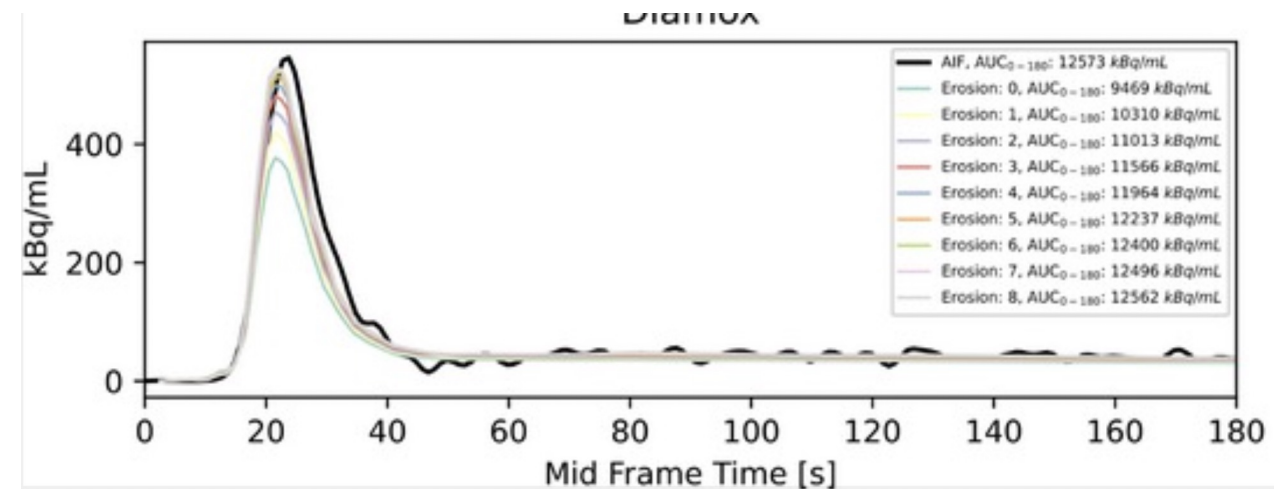
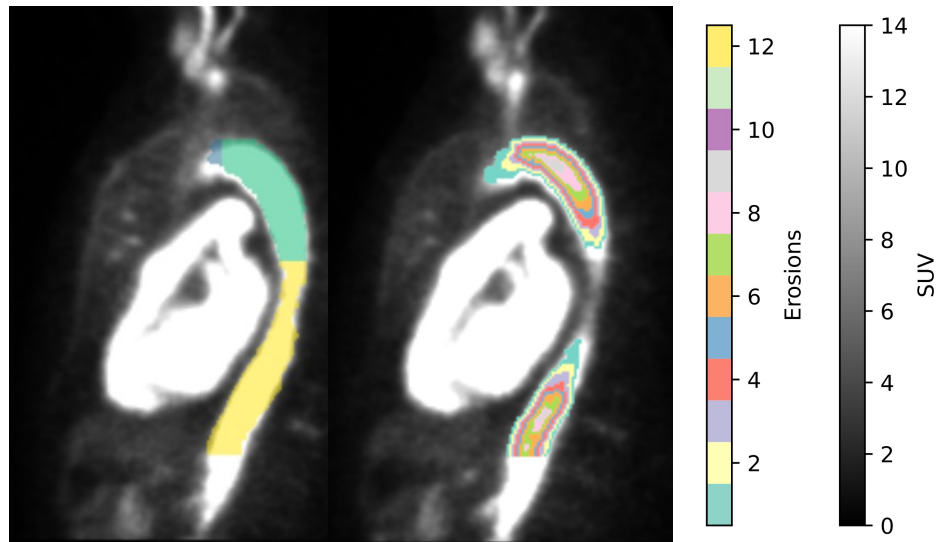




Image vs artery derived input function

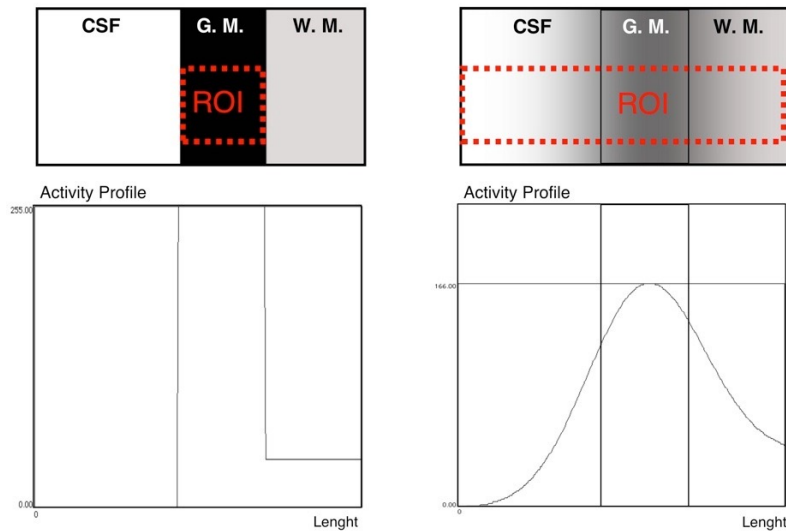
- AI based Aorta segmentation of low dose CT
- Erosion 8-10 mm
- Sample activity in centre 1- 4 mL in upper Aorta Descendens





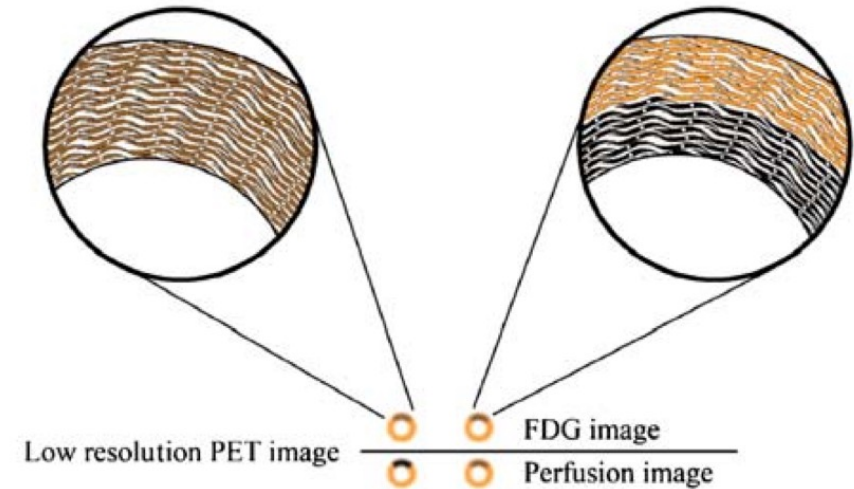
The Partial Volume Effect – resolution is limited

A Ideal Resolution B PET Scanner



Many tissues in ROI (tissue heterogeneity)
 Spill-in/out

Wall motion recovery (+) Wall motion recovery (-)





Two classes of models – wash in and wash out

$$C_t(t) = f C_a(t) \otimes e^{-(f/Vt) t};$$

f ($E \approx 1$) = $K_1 \Rightarrow K_1$ determines Perfusion information

Very Partial Volume sensitive

More Stable

$$C_t(t) = f \alpha C_a(t) \otimes e^{-(f/p) t}$$

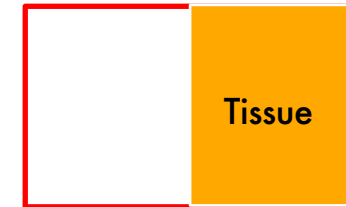
$$f = K_1 / \alpha ; K_1 = p k_2 , f = p k_2 / \alpha$$

$\Rightarrow k_2$ determines Perfusion information

Not Very Partial Volume sensitive

More Noisy

ROI



$$\alpha = 0.5 \text{ g/ml}$$

$$K_1 = 1.0 \text{ min}^{-1}$$

$$f = 2.0 \text{ ml g}^{-1} \text{ min}^{-1}$$

α Perfusable tissue fraction

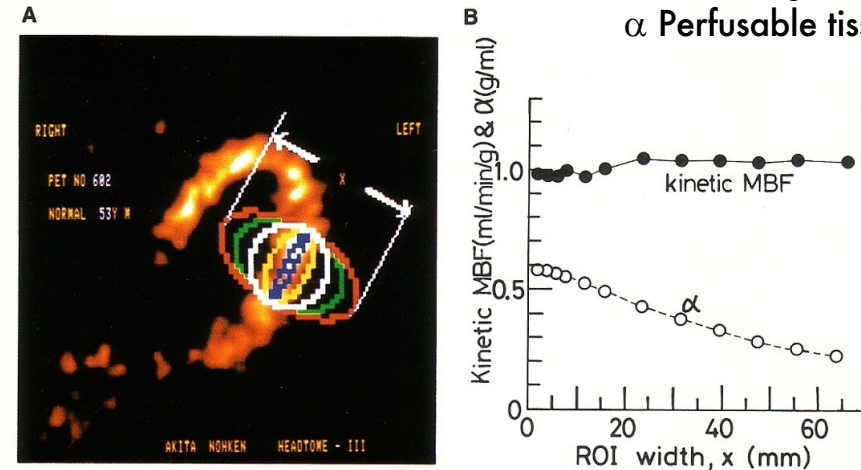
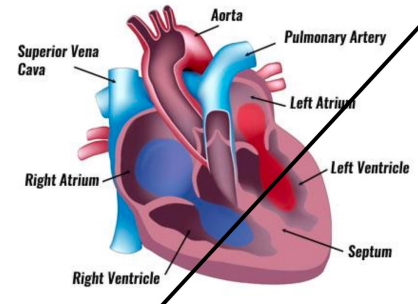
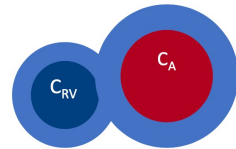
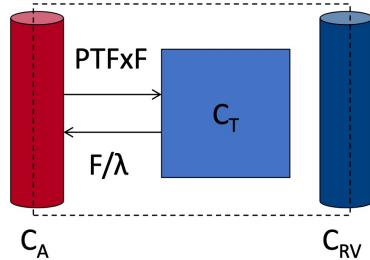


FIGURE 6. Various regions of interest (ROIs) in the myocardium of a normal volunteer (N3) (Panel A). Myocardial blood flows (MBFs) were calculated according to the present kinetic method for these ROIs. ROI size dependence of the calculated MBF and α obtained by the kinetic method (Panel B). The calculations were made for ROIs indicated in left panel. Calculated MBF values revealed no significant changes, while the calculated tissue fractions were decreased with increasing ROI size.



Myocardial blood flow (MBF) with $[^{15}\text{O}]\text{H}_2\text{O}$

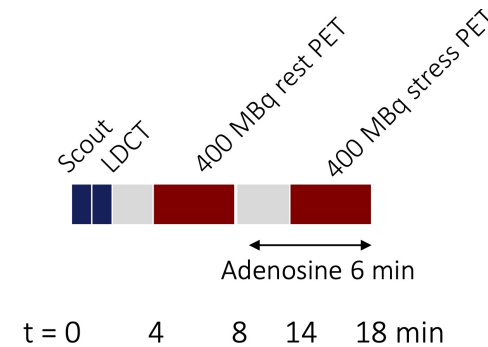
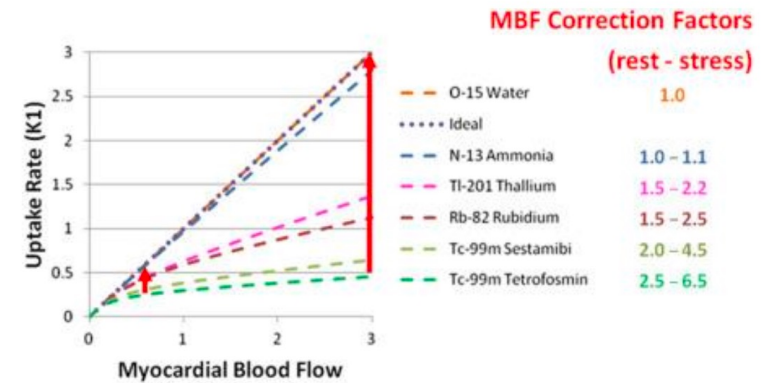
- Linear relationship between K1 and MBF
- Remove right and left ventricle BV in model



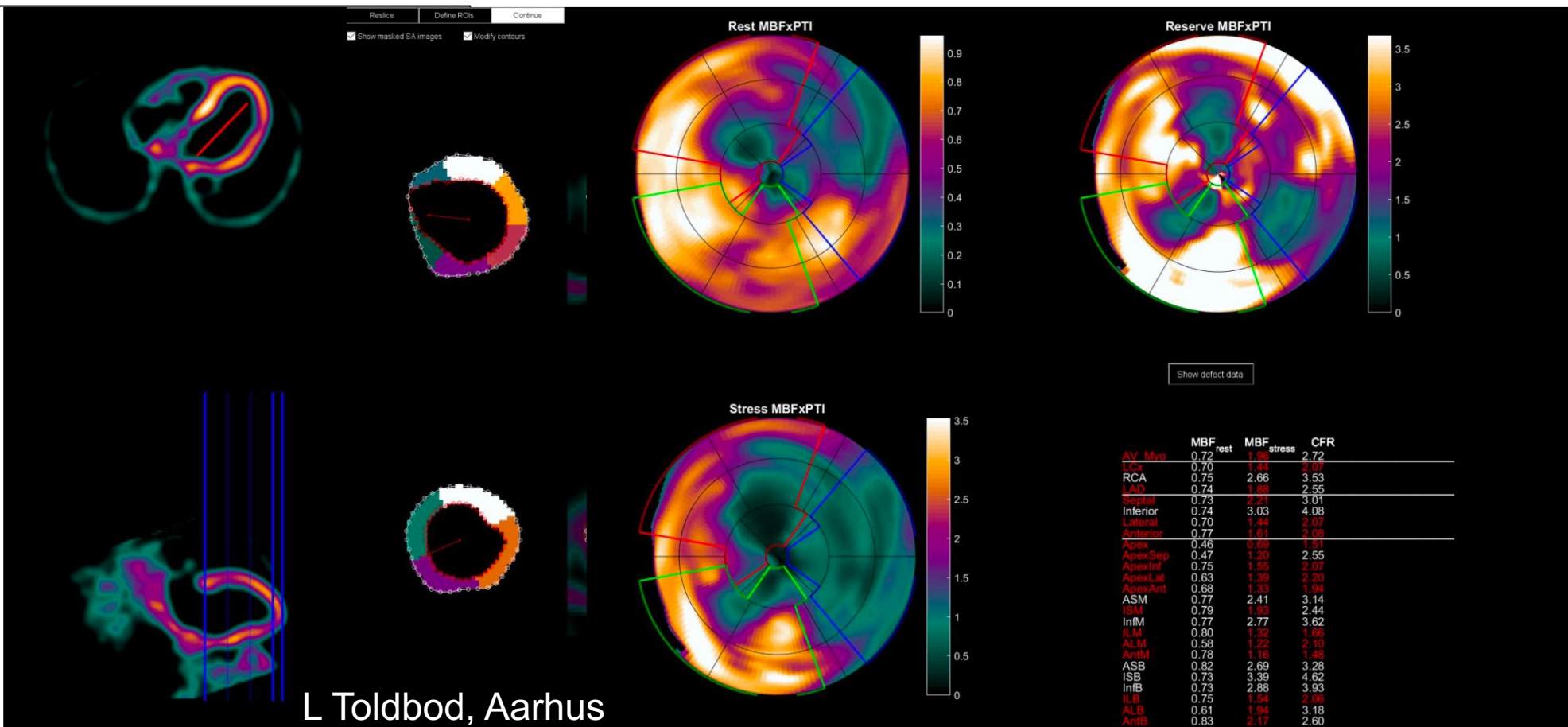
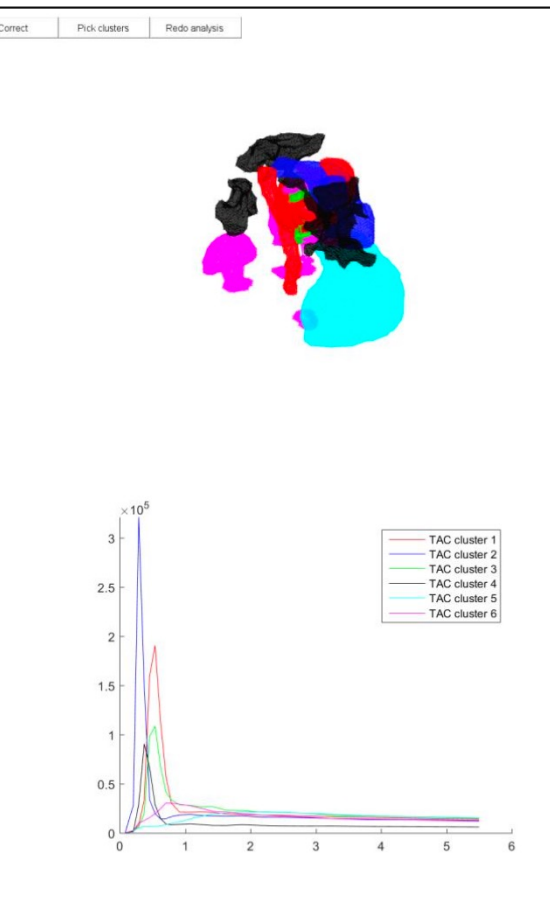
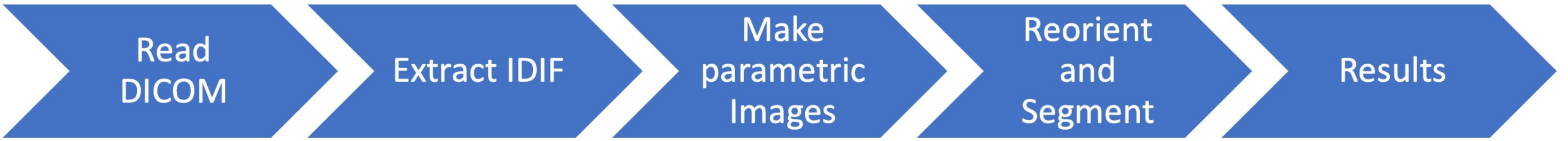
$$C_T(t) = PTF \cdot F \cdot C_A(t) \otimes e^{-F/\lambda t}$$

$$C_{PET}(t) = C_T(t) + V_A C_A(t) + V_V C_{RV}(t)$$

Uptake Rate



Analyse

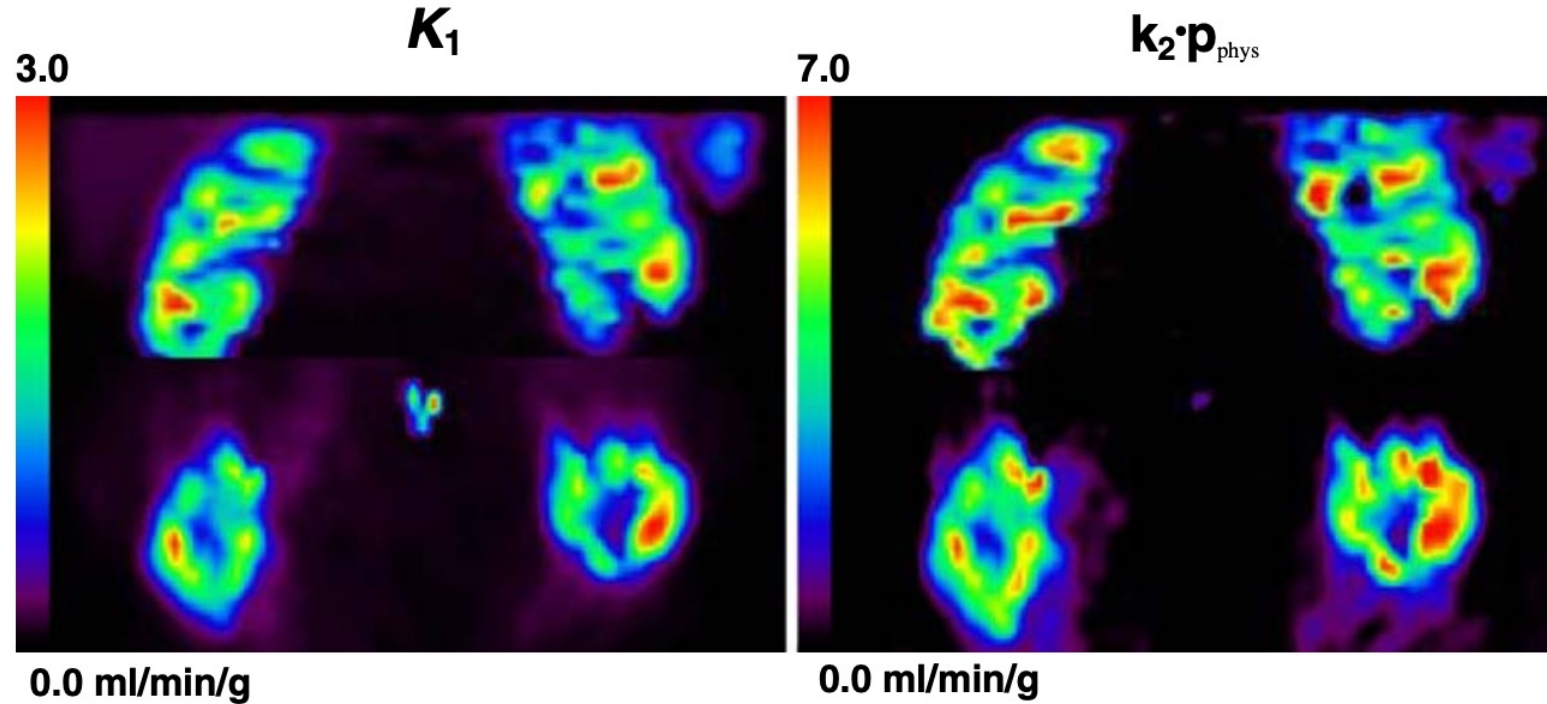




Renal perfusion with [¹⁵O]H₂O

- Limited studies

$$C_t(t) = C_a(t) V_b + (1 - V_b) K_1 C_a(t) \otimes e^{-(K_1/V)t}$$





Regional measurement of canine skeletal muscle blood flow by positron emission tomography with $H_2^{15}O$

ALAN J. FISCHMAN,^{1,2,4} HONGBING HSU,^{1,4} EDWARD A. CARTER,^{2,3,5}
YONG M. YU,^{2,3,5} RONALD G. TOMPKINS,^{2,3,5} J. LUIS GUERRERO,³
VERNON R. YOUNG,^{2,3} AND NATHANIEL M. ALPERT¹

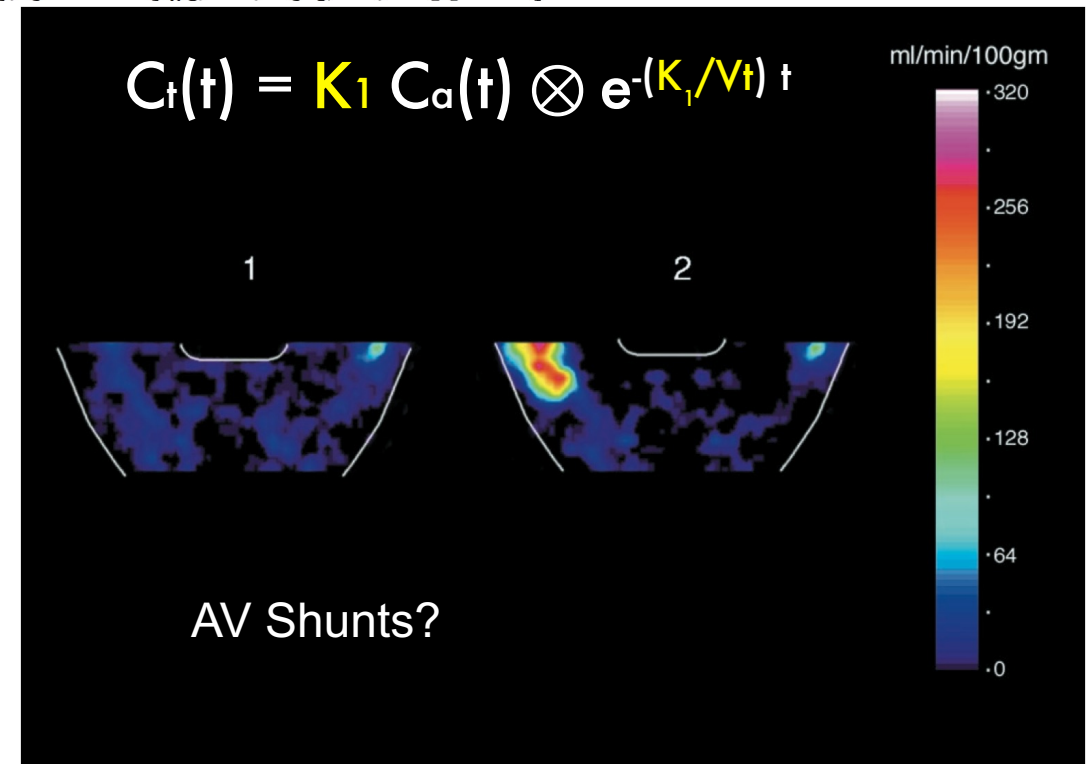
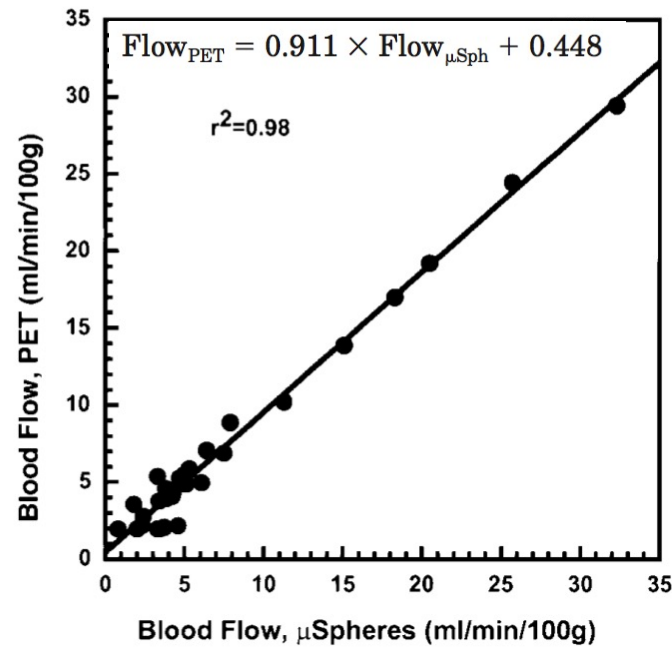


Fig. 7. Examples of blood flow maps (coronal projection) derived by pixel-by-pixel solution of Eq. 6. 1. Low flow, baseline map. 2. High-flow map derived after local infusion of adenosine in the right thigh. Blue area in upper right of both panels represents bone marrow.



Eur J Nucl Med Mol Imaging (2008) 35:1899–1911
DOI 10.1007/s00259-008-0796-z

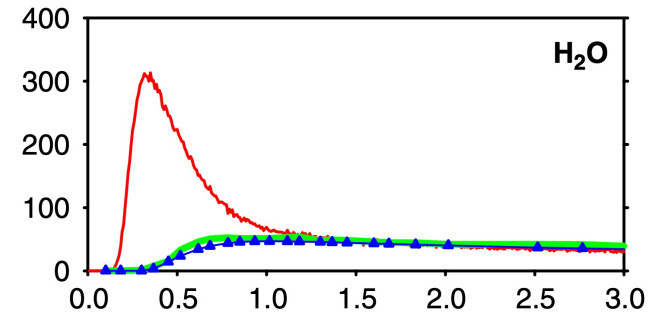
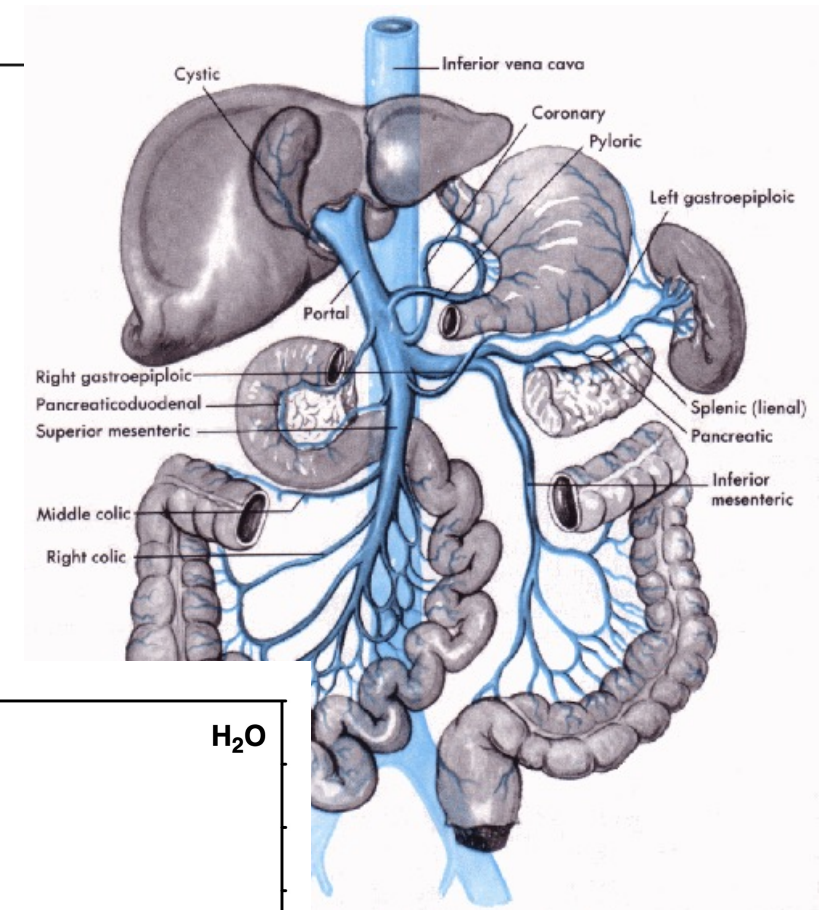
ORIGINAL ARTICLE

Non-invasive estimation of hepatic blood perfusion from $H_2^{15}O$ PET images using tissue-derived arterial and portal input functions

N. Kudomi · L. Slimani · M. J. Järvisalo · J. Kiss ·
R. Lautamäki · G. A. Naum · T. Savunen · J. Knuuti ·
H. Iida · P. Nuutila · P. Iozzo

Possible, but complicated

- dual liver input (1/3 hepatic artery, 2/3 portal vein)
- non-invasive estimation of input from portal vein
- Portal vein is tissue curve from intestines

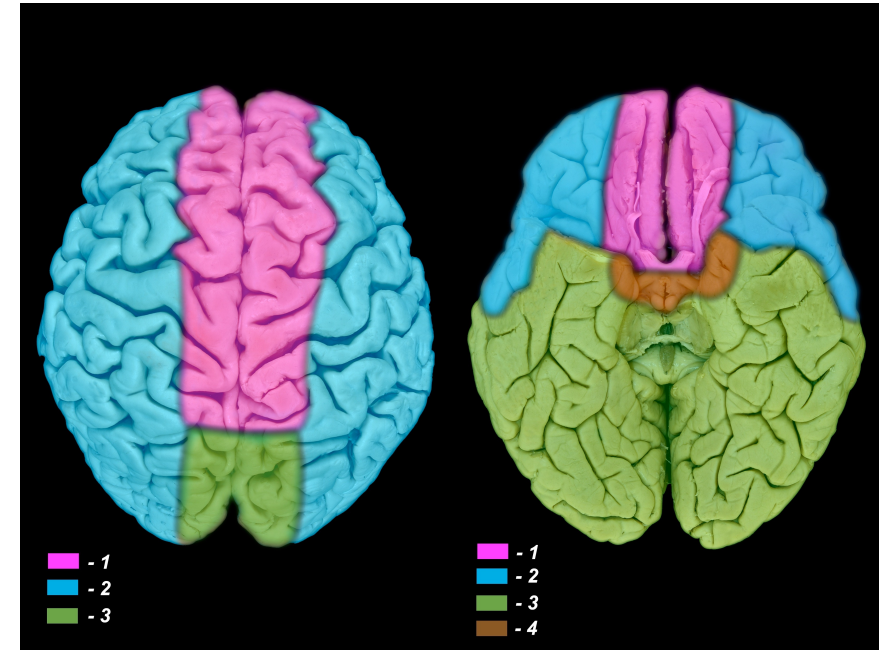
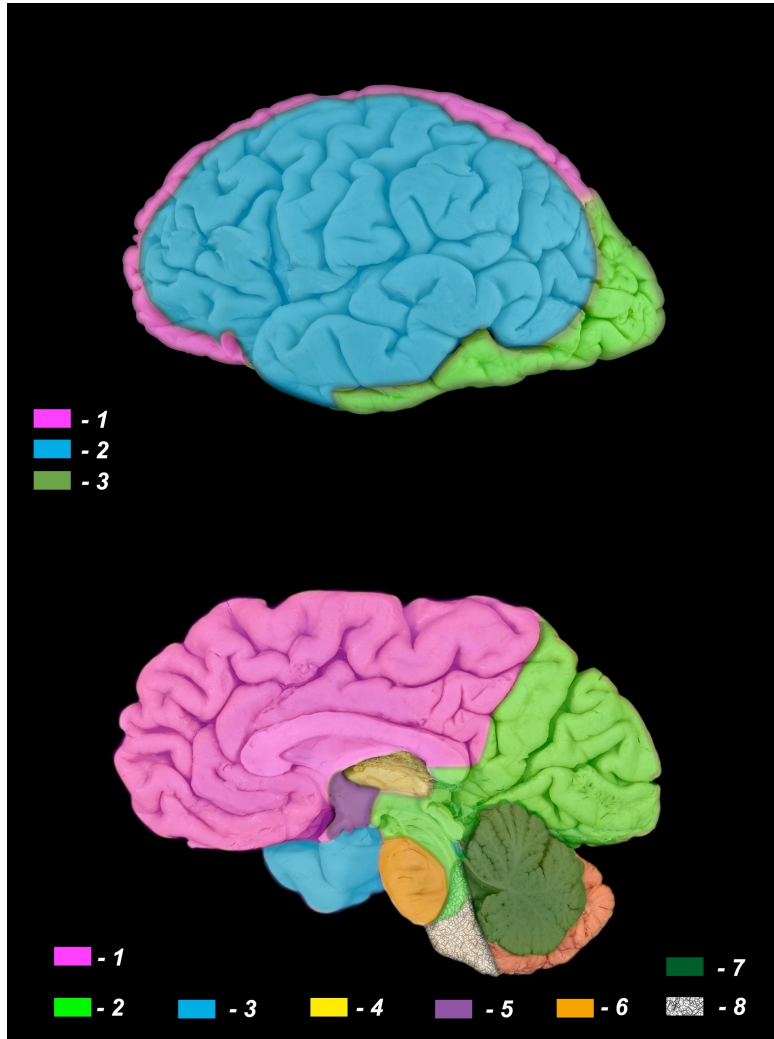


Clinical use of CBF

The Diamox test



Vascularisation of the cerebral cortex



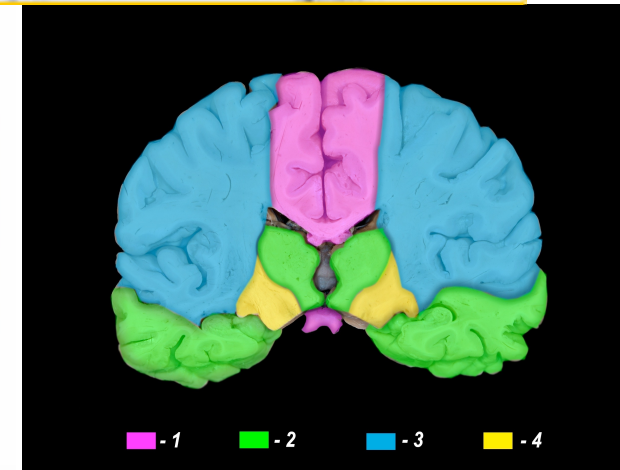
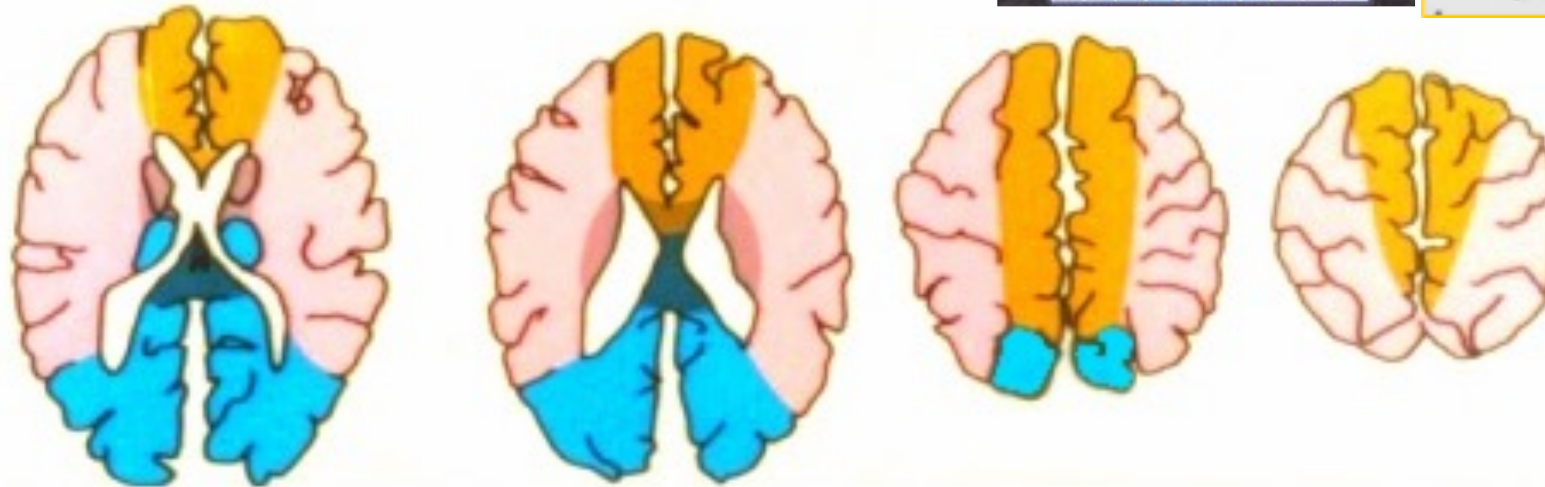
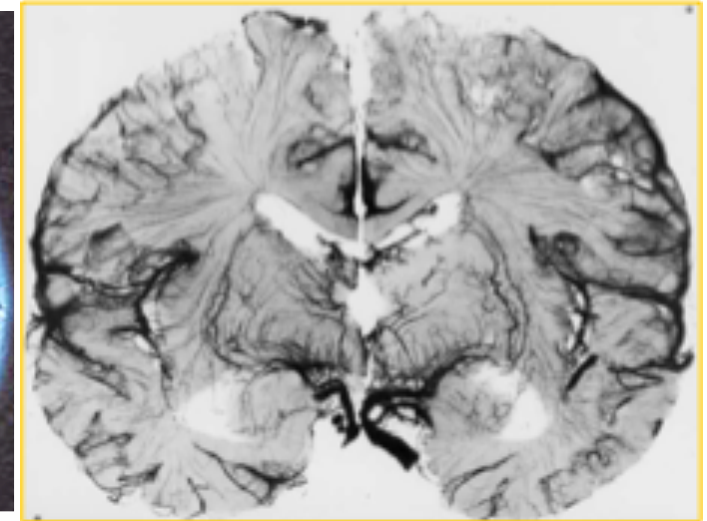
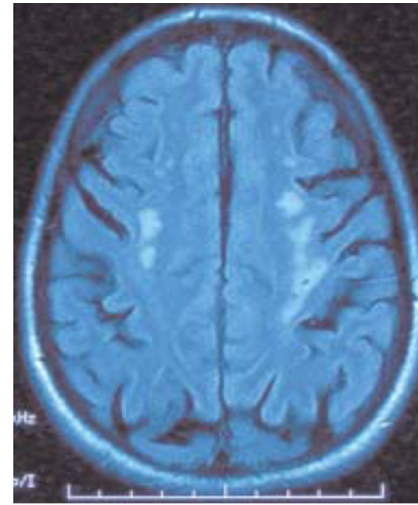
Borderline regions are associated with the potentially greater risk of ischaemia leading to brain stroke

„watershed infarct”



Small arterial vessels:

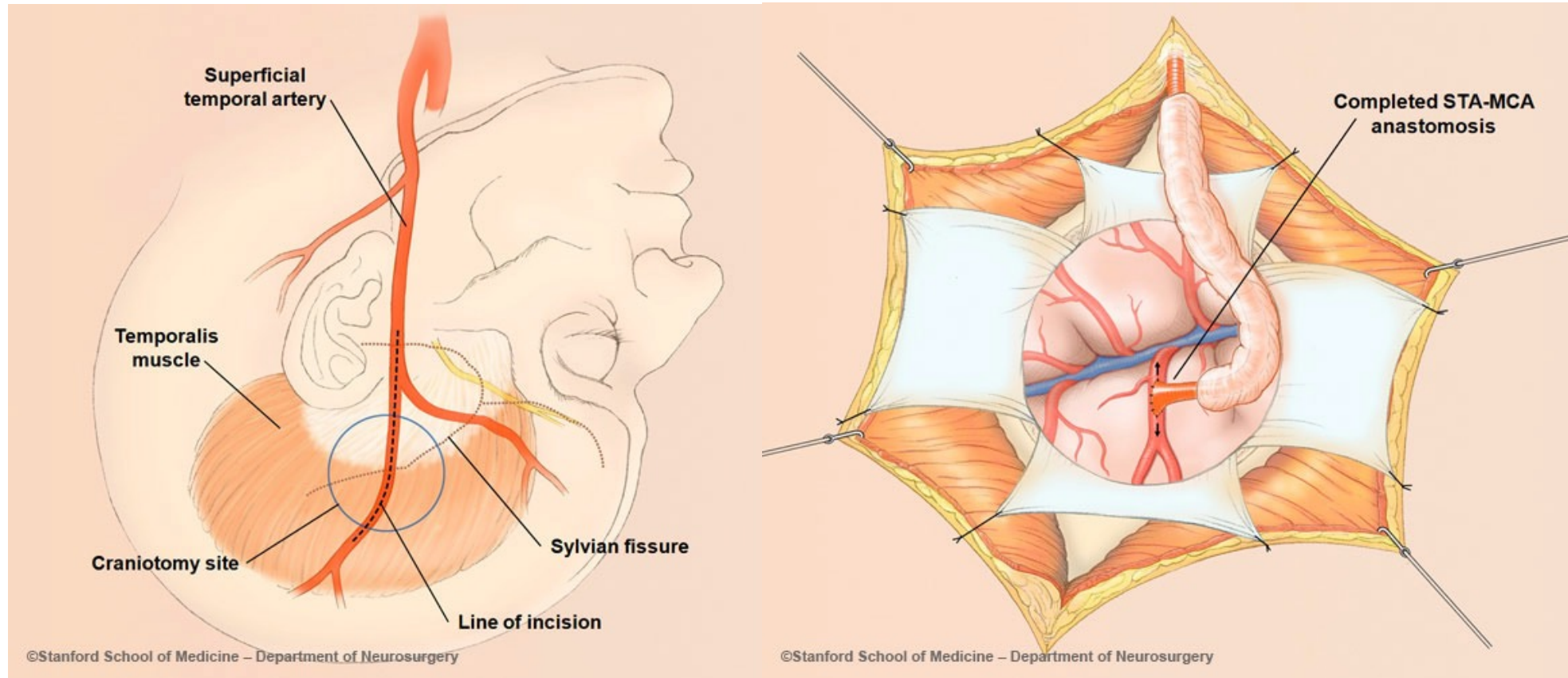
- Deep (perforating) vessels
- functionally terminal vessels
do not form collateral circulation





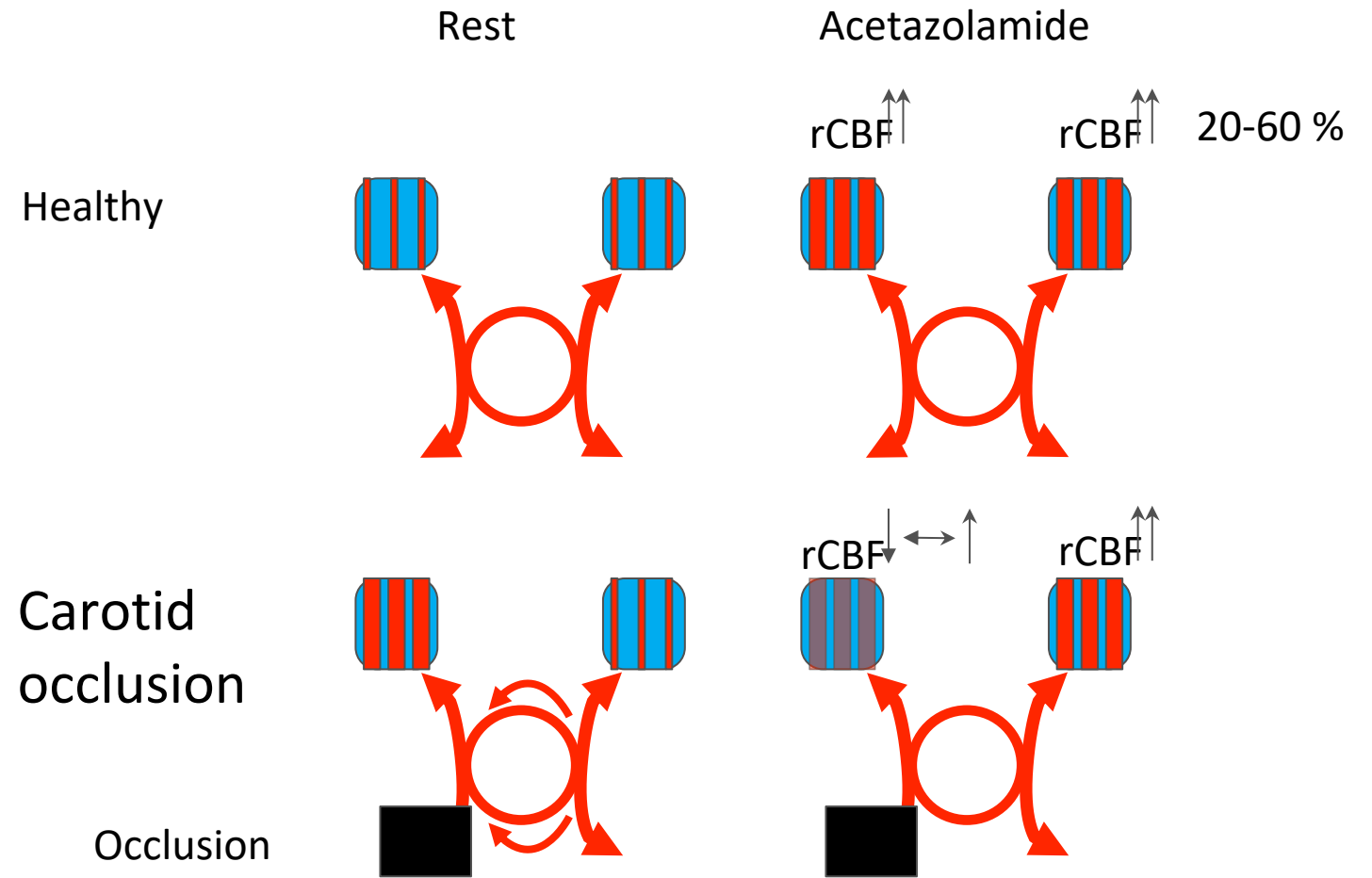
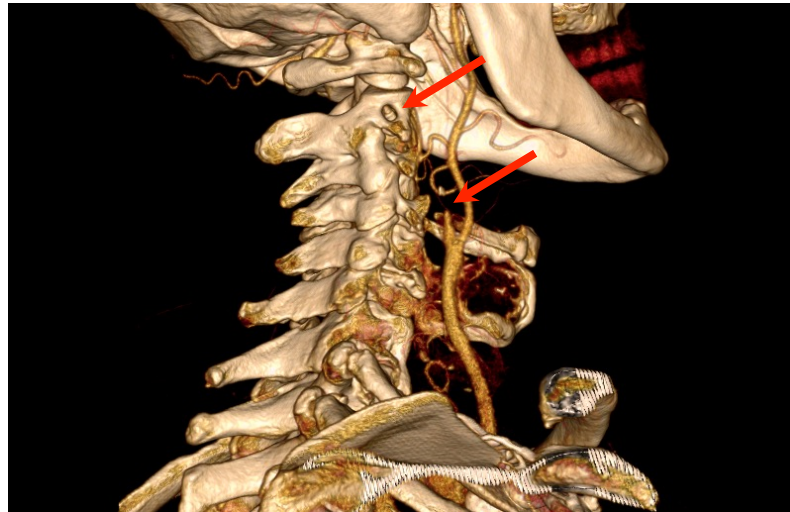
Treating internal carotid occlusions: EC-IC bypass

Direct superficial temporal artery (STA) to middle cerebral artery (MCA) bypass





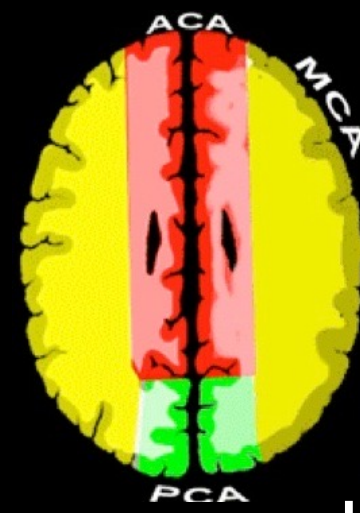
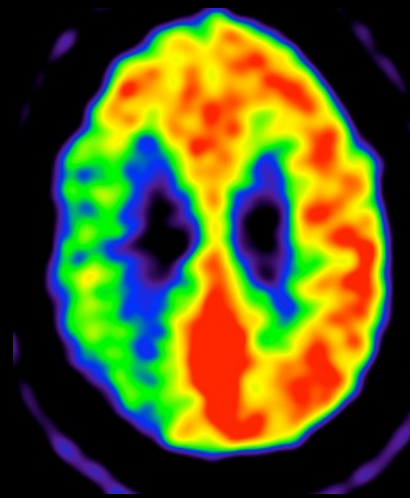
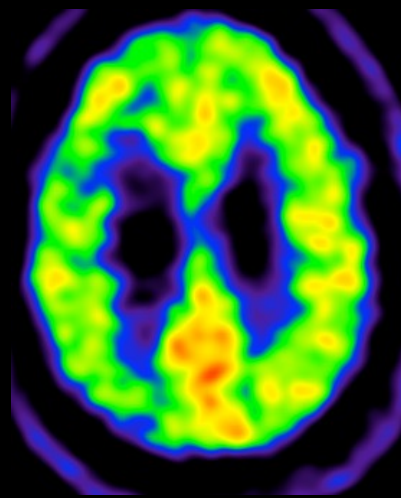
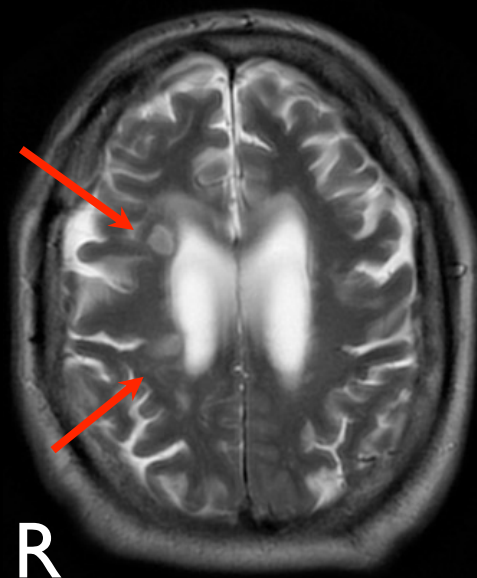
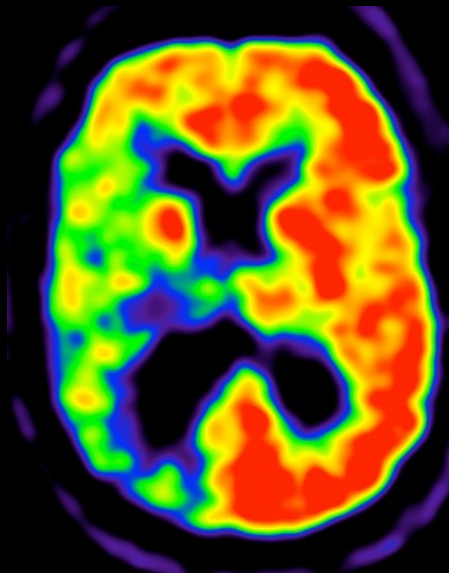
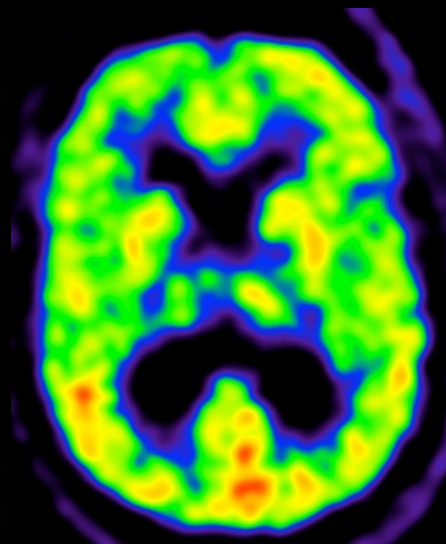
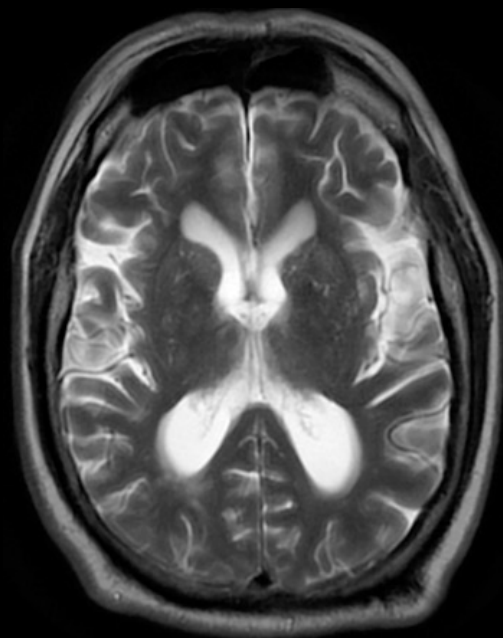
The reverse Robin Hood Syndrome



T2 MRI

rCBF Baseline

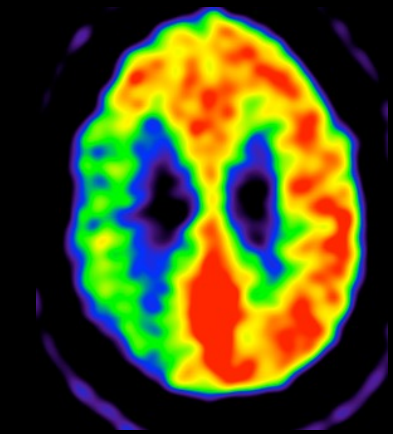
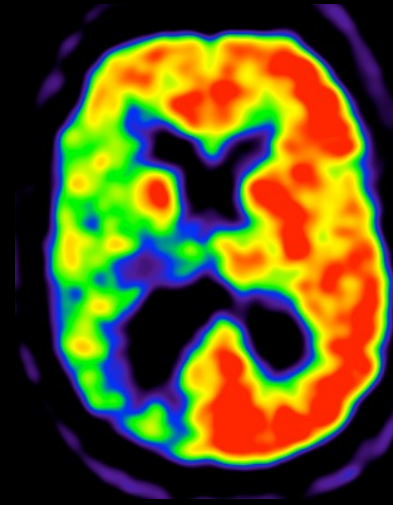
rCBF Diamox



Follow-up

- Refused surgery
- 2 months after PET
 - Pneumonia
 - Septicaemia
 - Hypovolemia
 - Hypotension
 - R Stroke

rCBF Diamox



CT 2 mth post PET





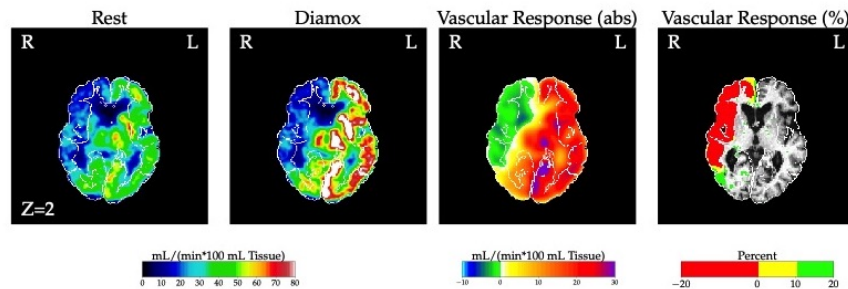
Standard Diamox report



Klinik for Klinisk Fysiologi, Nuklear medicin og PET
 Rigshospitalet
 Blegdamsvej 9
 2100 København Ø

Cerebrovascular hemodynamic response test with [¹⁵O]H₂O PET

Patient name	CPR	Age	Sex	Scan date	Weight (kg)	Dose (MBq)
					72	800

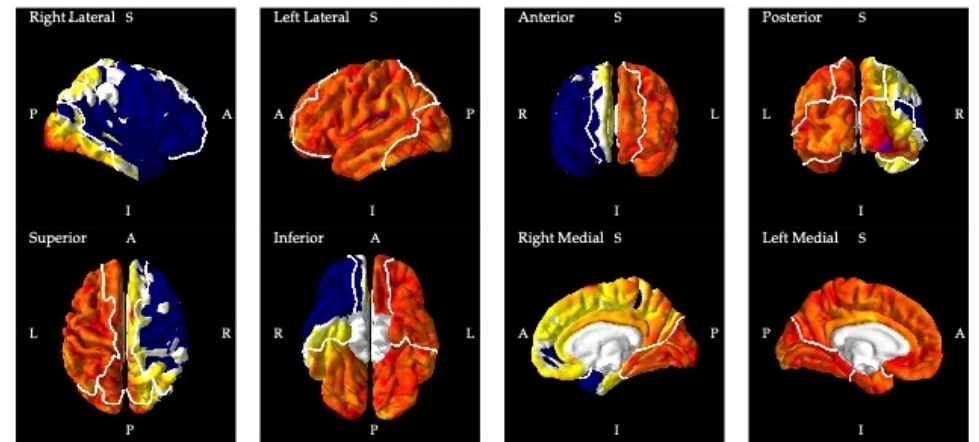
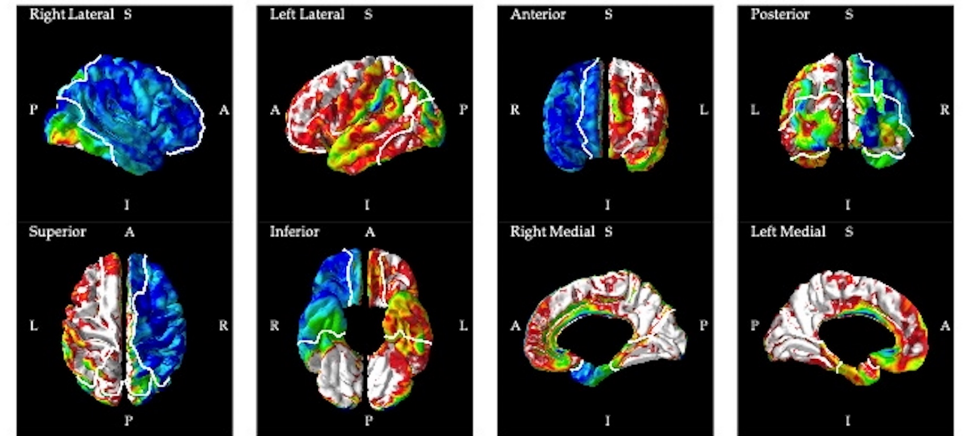


Vascular territories	Volume (mL)	Cortical regional cerebral blood flow (mL/(min*100 mL Tissue))					
		Left			Right		
		Rest	Diamox	Change (%)	Rest	Diamox	Change (%)
ACA	184	41.4	66.7	61.3	24.2	25.4	5.0
Frontal MCA	99	42.0	67.8	61.7	17.7	15.9	-10.2
Parietotemporal MCA	335	36.8	58.6	59.1	18.8	17.3	-8.3
PCA	135	39.8	64.0	60.9	34.3	45.7	33.1
Watershed areas							
Frontal MCA/ACA	11	47.5	76.5	60.9	16.8	16.5	-1.8
Parietotemporal MCA/ACA	32	41.6	67.2	61.6	17.4	18.0	3.2
MCA/PCA	19	39.1	63.9	63.4	22.4	20.3	-9.3
ACA/PCA	14	42.8	69.7	63.0	31.7	39.3	23.9
Anatomical structure							
Cerebellum	108	34.9	55.9	60.1			
Threshold defined areas in cortical regions							
Steal		190		-8.5			
<10%		44		4.3			
<20%		33		14.9			

rCBF Change= (Diamox-Rest)*100 /Rest (%)
 Rest is based on 1 PET measurement:
h_{in}-45f-5min-K1
 Diamox is based on 1 PET measurement:
diamox-45f-5min-K1

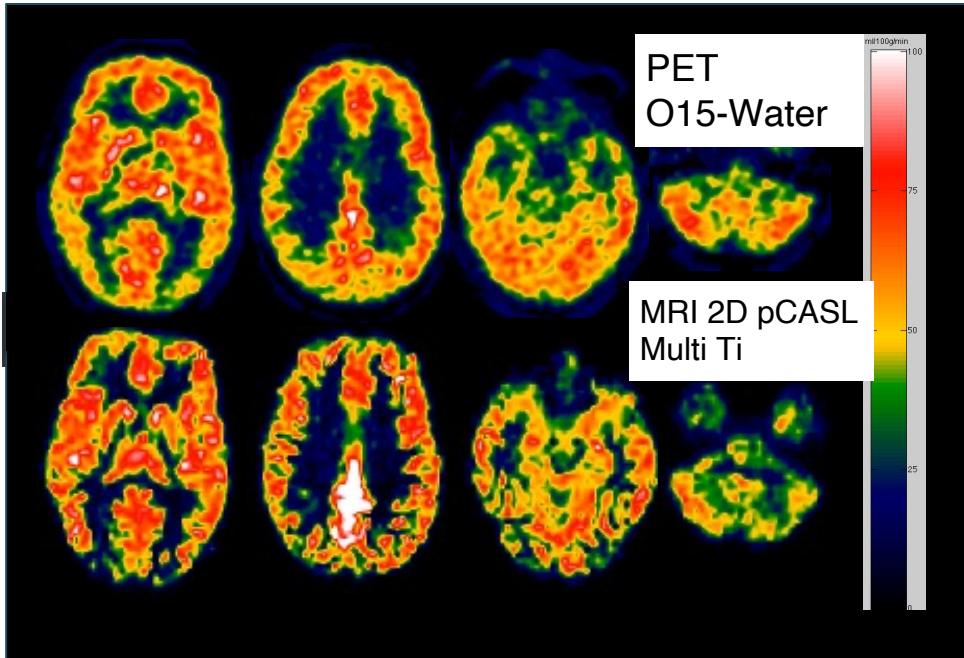
The arterial territories are defined according to Tatu L et al., Neurology. 1998;50:1699-1708.

rCBF during Diamox

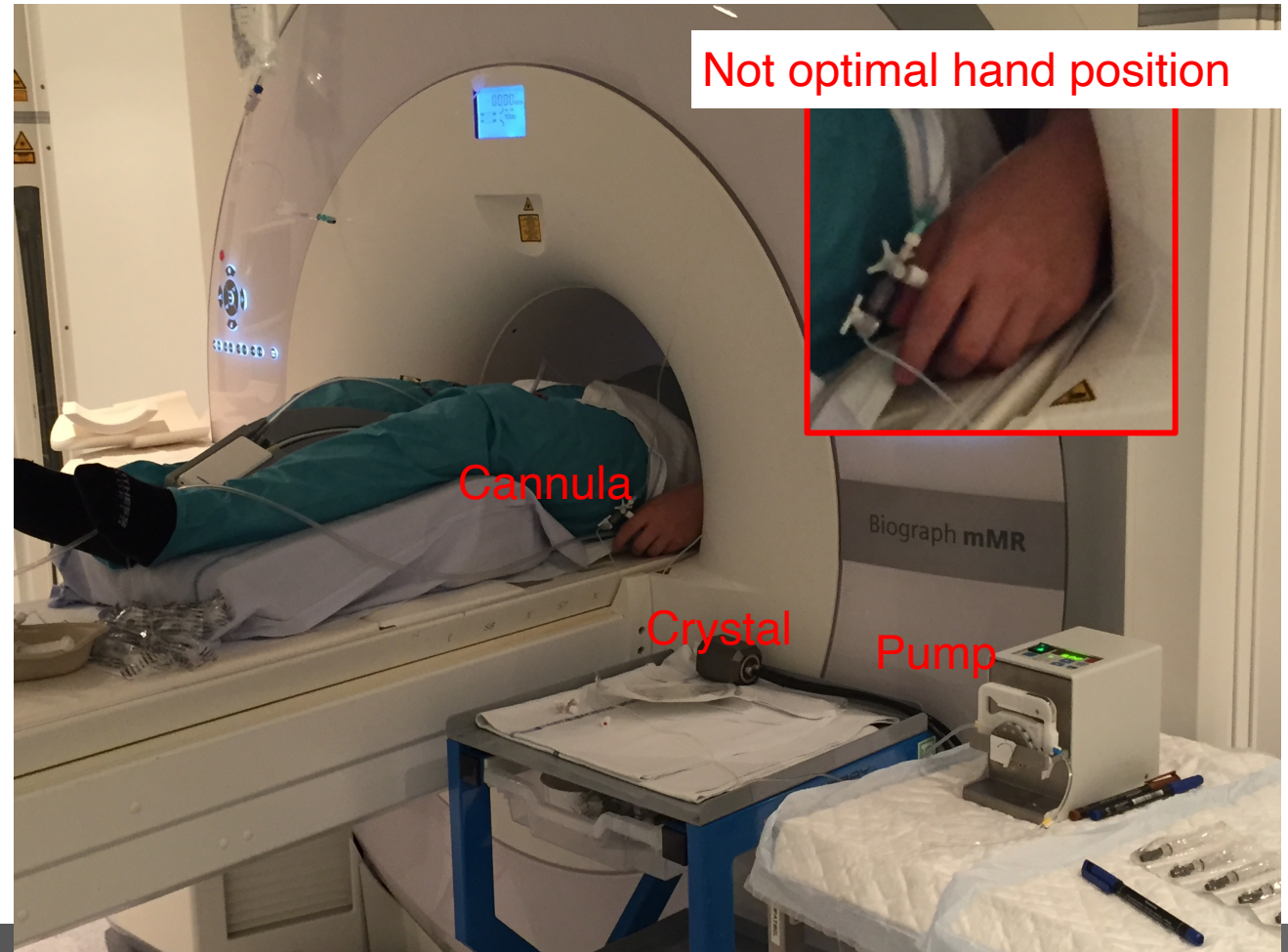




Regional Cerebral Blood Flow PET/MRI O15-Water & Arterial Spin labelling (ASL)

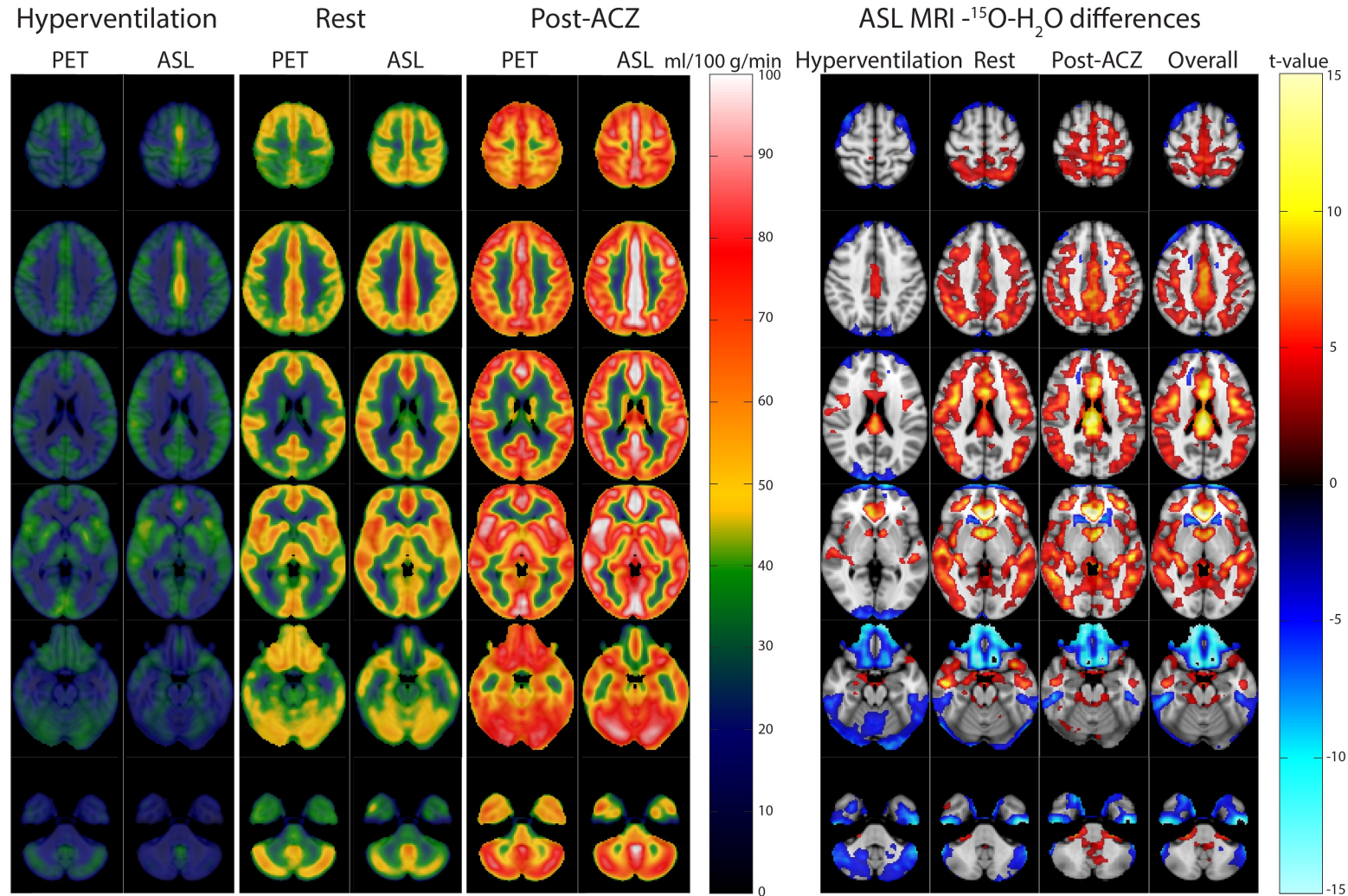


Simultaneous
Measurement
Rest

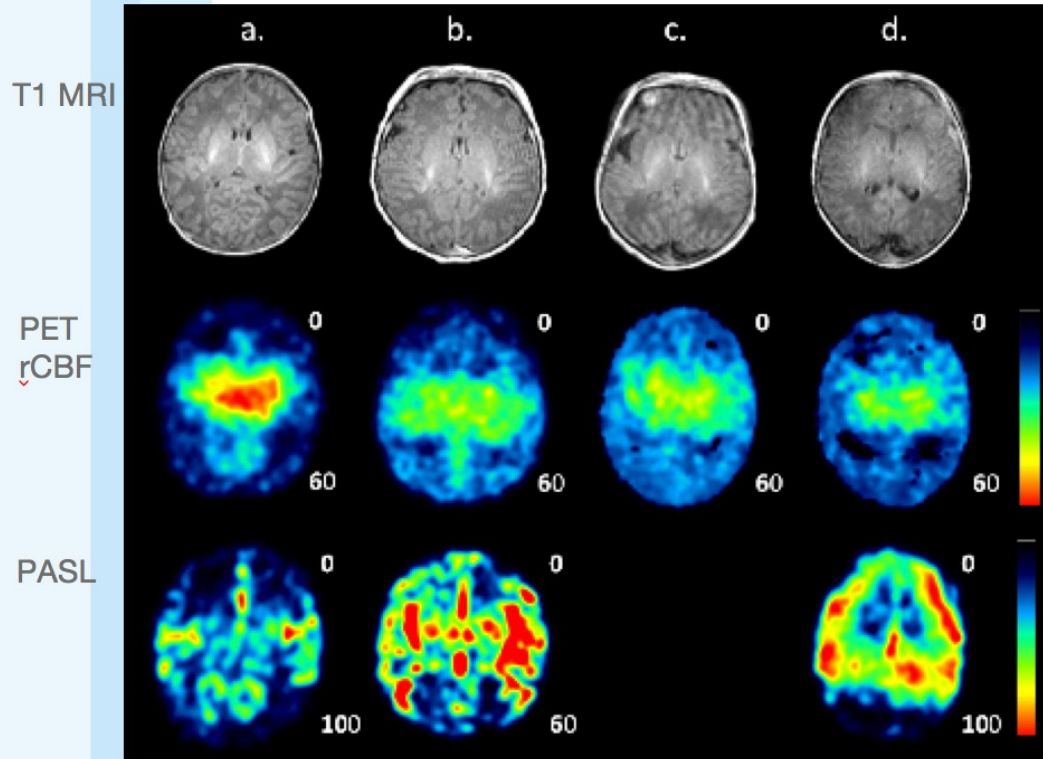




Average ASL and PET CBF images in different perfusion states and differences between techniques



Brain PET/MRI O15-H2O rCBF ASL i healthy newborn children



T1 MRI

PET
rCBF

PASL

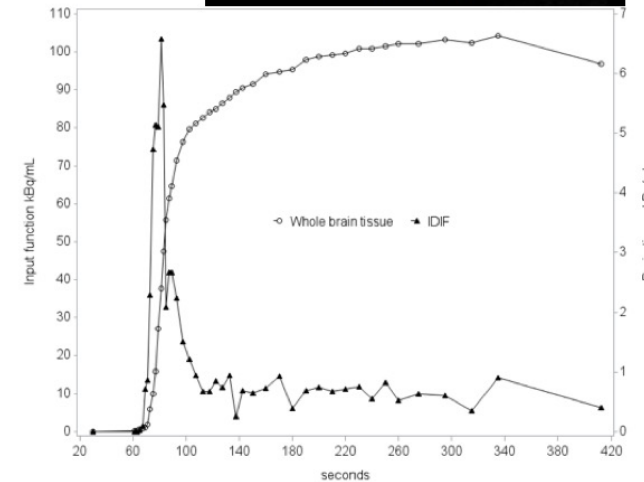
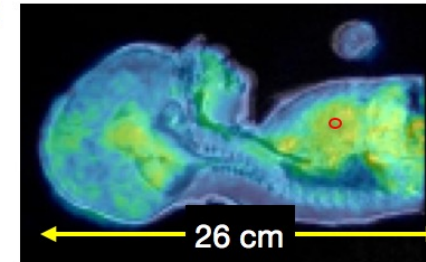
rCBF (ml/100g/min)

	a	b	c	d	Mean (SD)
WM unmyel	10.1	11.5	10.1	9.3	10.3 (0.9)
WM myel	23.3	15.9	14.6	10.6	16.1 (5.3)
Thalami	42.9	30.3	29.3	22.5	31.3 (8.5)
Whole brain	22.2	17.8	16.2	15.0	17.8 (3.1)

Andersen JB, et al

4 Healthy Newborn Children
Quantitative rCBF
Input: Left ventricle
14 MBq O15 H2O = 0.3 mSv
Simultaneous PASL
AC-RESOLUTE

Clinical Perspective:
Periventricular Leukomalacia (PVL)
Moya-Moya



Rigshospitalet

Klinik for Klinisk Fysiologi, Nuklear Medicin & PET

KØBENHAVNS UNIVERSITET
DET SUNDHEDSVIDENSKABELIGE FAKULTET



REGION

