

# Receptor kinetics

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# Overview

- What goes on in the blood?
- What goes on in the brain?
  - Some useful concepts from biochemistry
- Kinetic modeling of PET / SPECT data
- Compare outputs from our kinetic models to in vitro analyses
- Emphasize on some assumptions – please don't violate.

# Established nomenclature for PET / SPECT

Journal of Cerebral Blood Flow & Metabolism (2007) 27, 1533–1539

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## Review Article

# Consensus nomenclature for *in vivo* imaging of reversibly binding radioligands

Robert B Innis<sup>1</sup>, Vincent J Cunningham<sup>2</sup>, Jacques Delforge<sup>3</sup>, Masahiro Fujita<sup>1</sup>, Albert Gjedde<sup>4</sup>, Roger N Gunn<sup>5</sup>, James Holden<sup>6</sup>, Sylvain Houle<sup>7</sup>, Sung-Cheng Huang<sup>8</sup>, Masanori Ichise<sup>9</sup>, Hidehiro Iida<sup>10</sup>, Hiroshi Ito<sup>11</sup>, Yuichi Kimura<sup>12</sup>, Robert A Koeppe<sup>13</sup>, Gitte M Knudsen<sup>14</sup>, Juhani Knuuti<sup>15</sup>, Adriaan A Lammertsma<sup>16</sup>, Marc Laruelle<sup>2</sup>, Jean Logan<sup>17</sup>, Ralph Paul Maguire<sup>18</sup>, Mark A Mintun<sup>19</sup>, Evan D Morris<sup>20</sup>, Ramin Parsey<sup>9</sup>, Julie C Price<sup>21</sup>, Mark Slifstein<sup>9</sup>, Vesna Sossi<sup>22</sup>, Tetsuya Suhara<sup>11</sup>, John R Votaw<sup>23</sup>, Dean F Wong<sup>24</sup> and Richard E Carson<sup>25</sup>



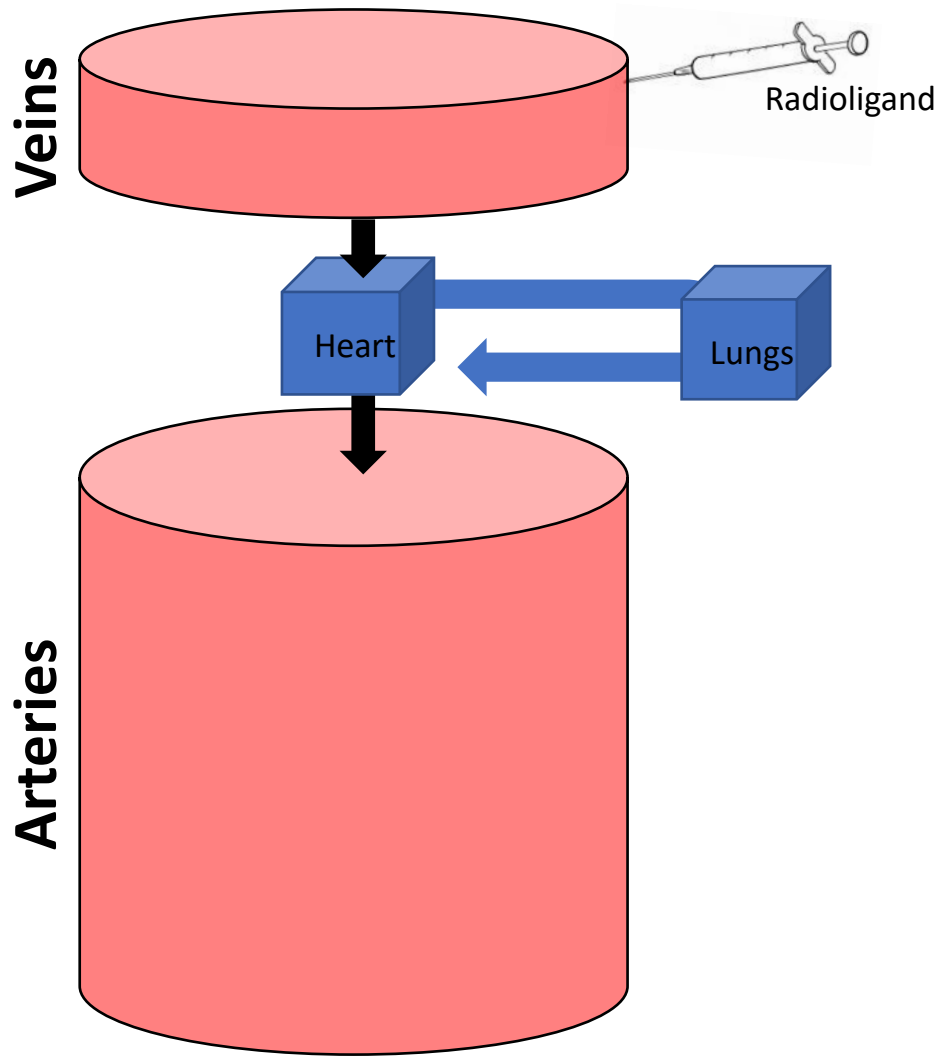
Martin Schain

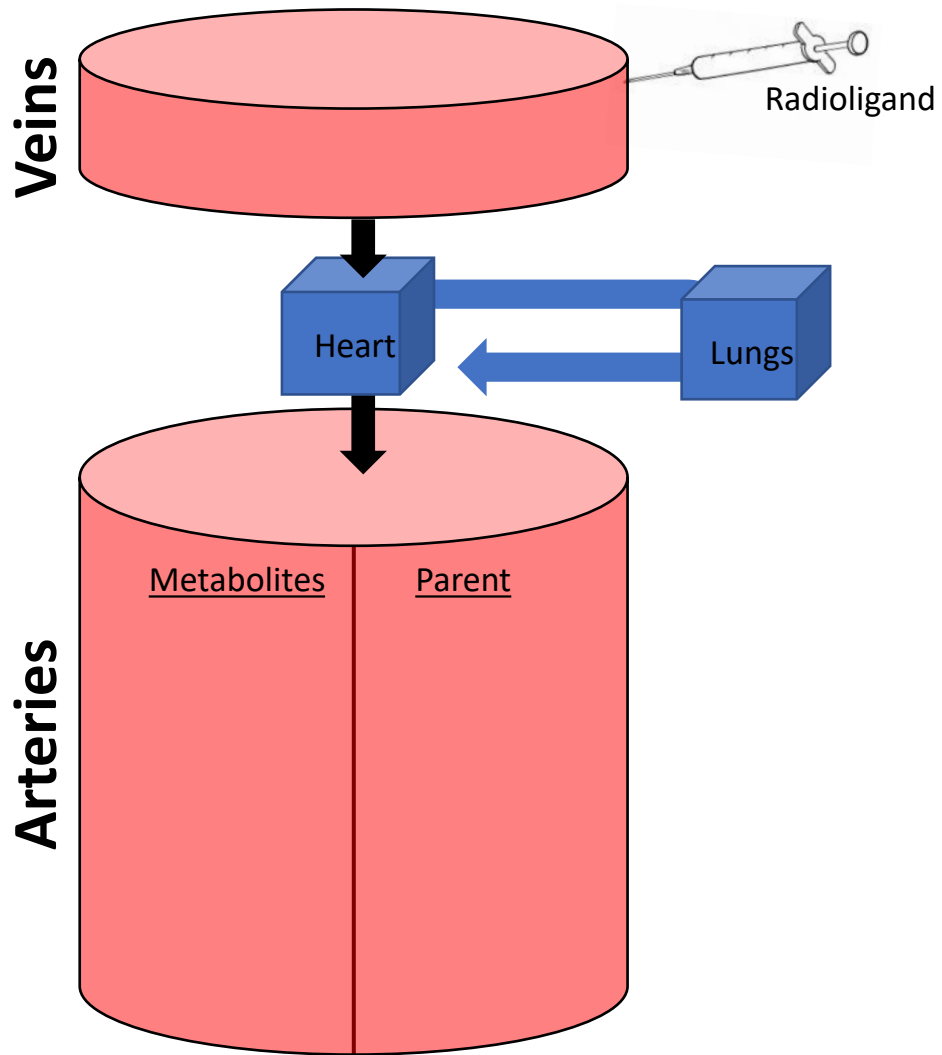
NRU, Copenhagen University Hospital, Rigshospitalet

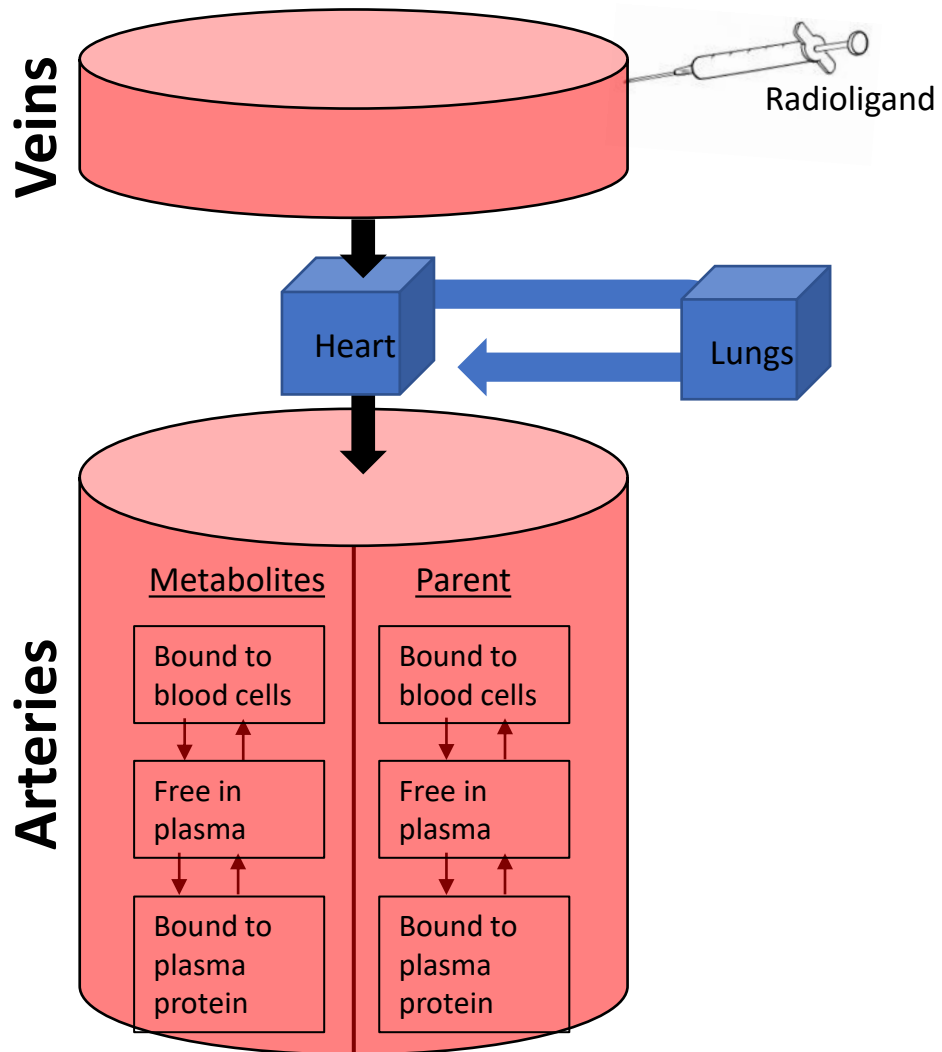


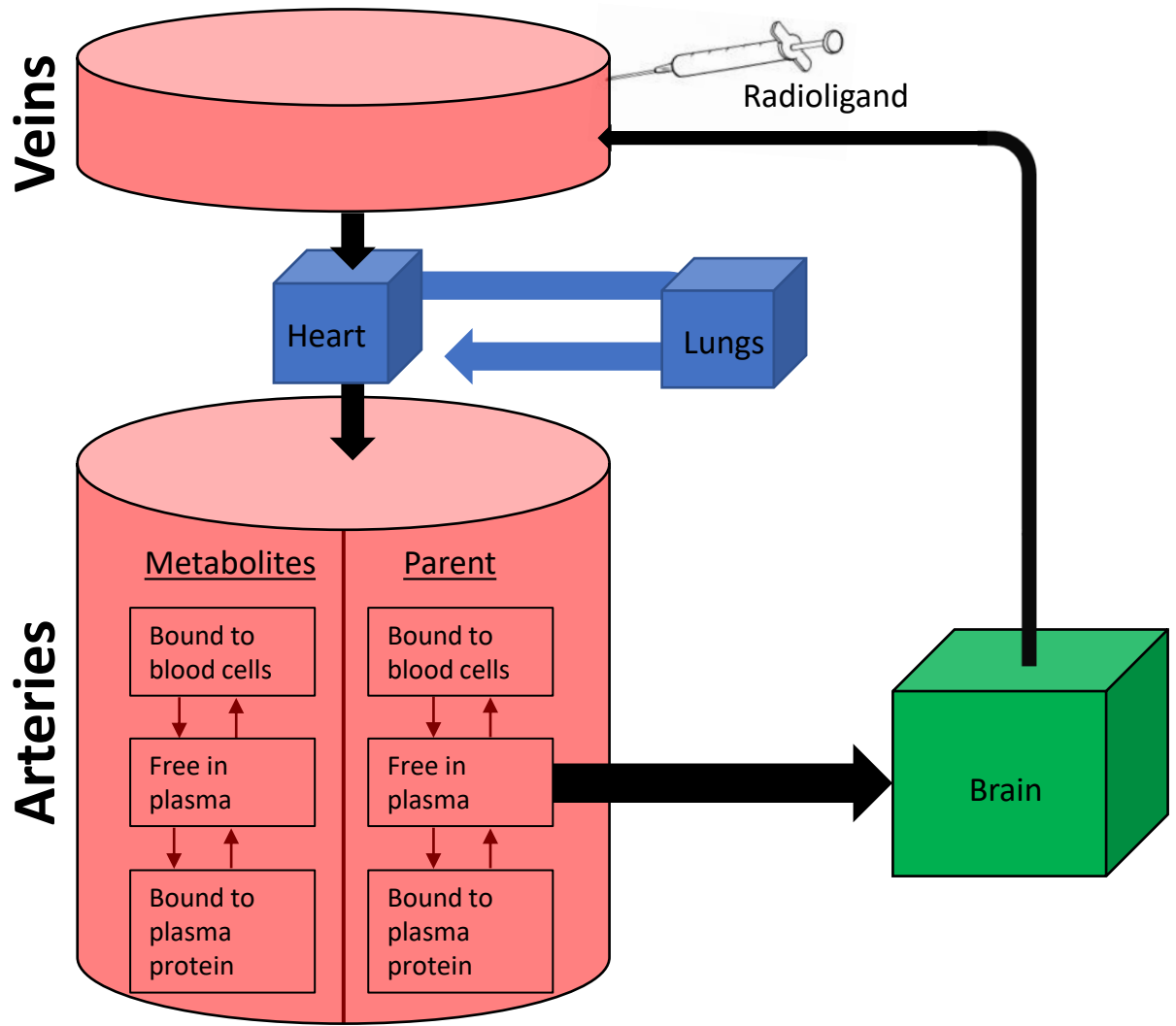
Antaros  
Medical

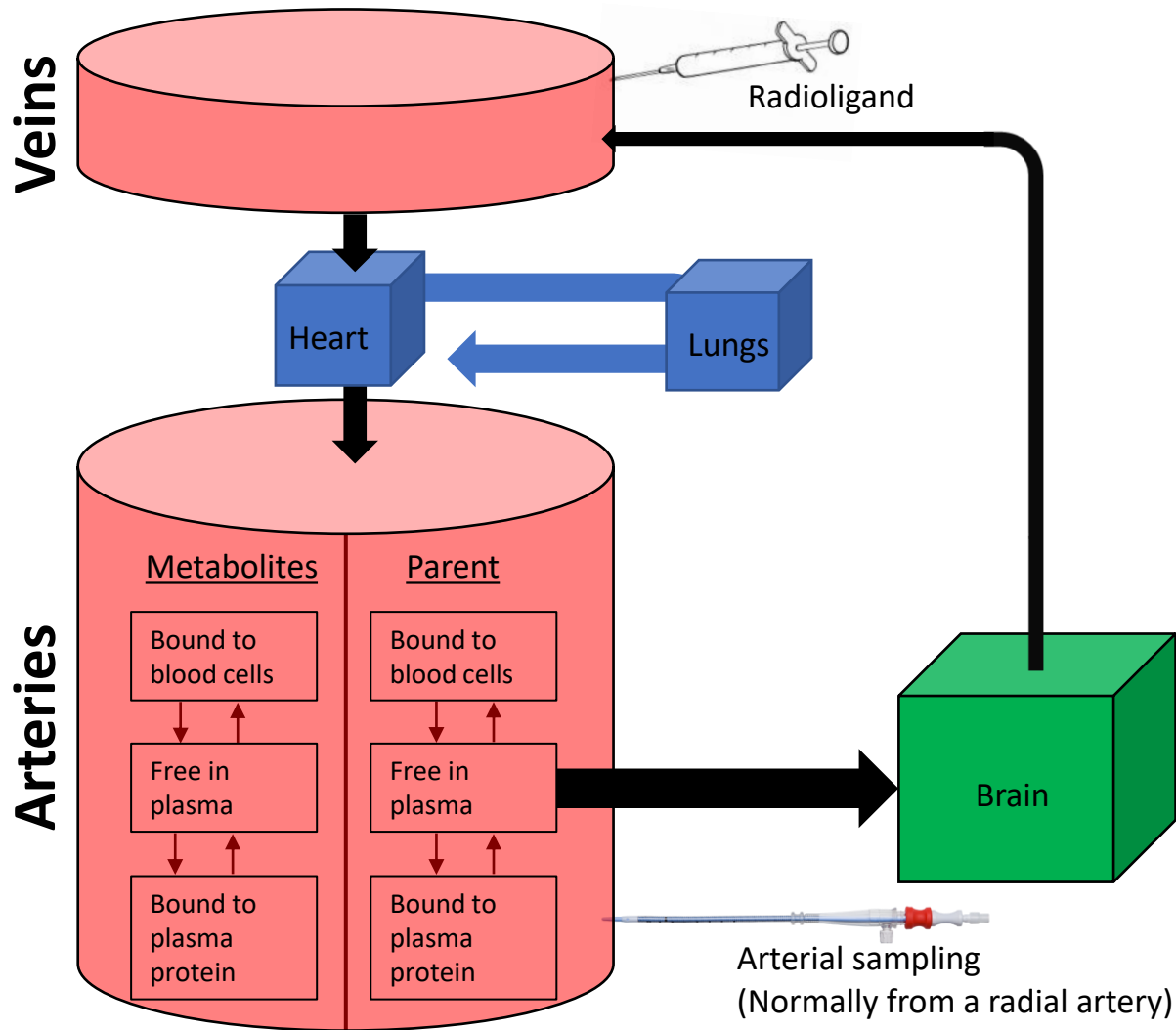












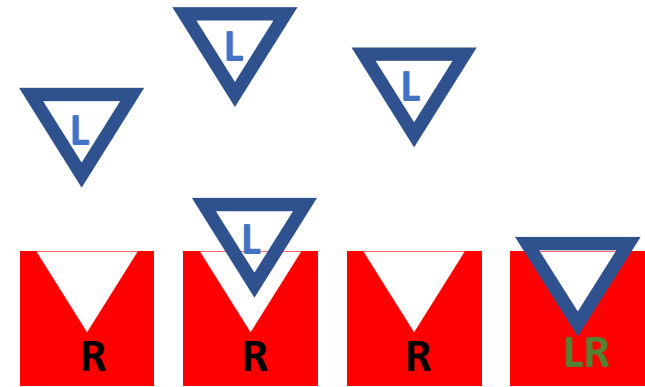
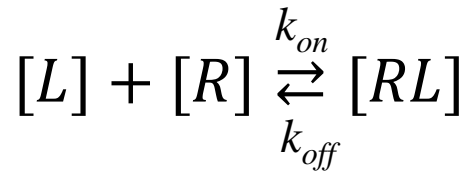


# Some useful concepts from biochemistry

## Single binding site model

[RL]: Conc. of bound receptor-ligand complexes

[L] : Conc. of free ligand

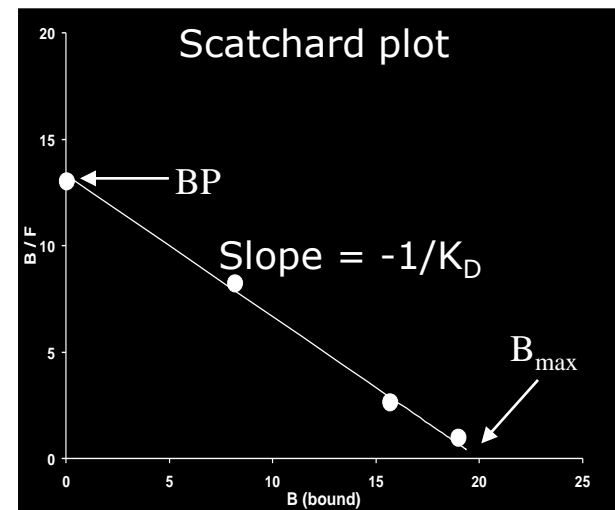


## Dissociation constant

$$K_D = \frac{k_{off}}{k_{on}}$$

## Binding potential

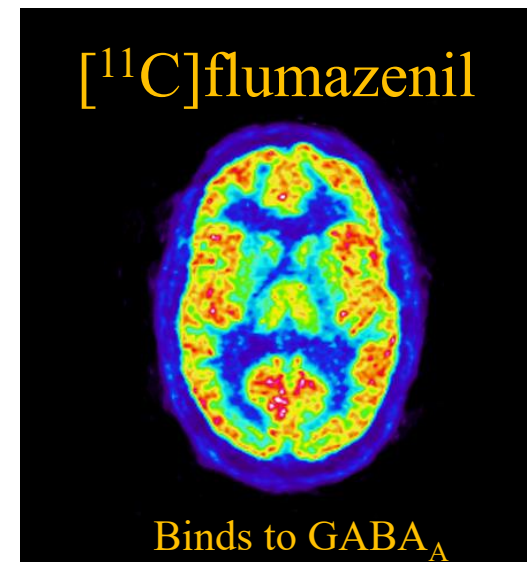
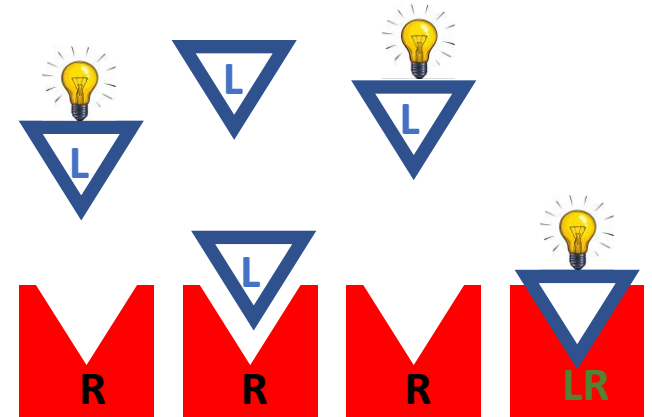
$$BP = \frac{[RL]}{[L]} = \frac{\text{"bound"}}{\text{"free"}}$$



# Some useful concepts from biochemistry

## Single binding site model

What would we ideally want from a PET experiment?

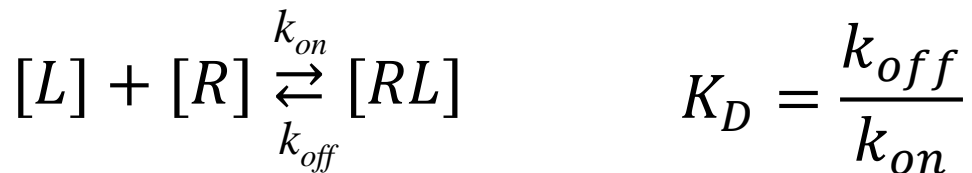


# Some useful concepts from biochemistry

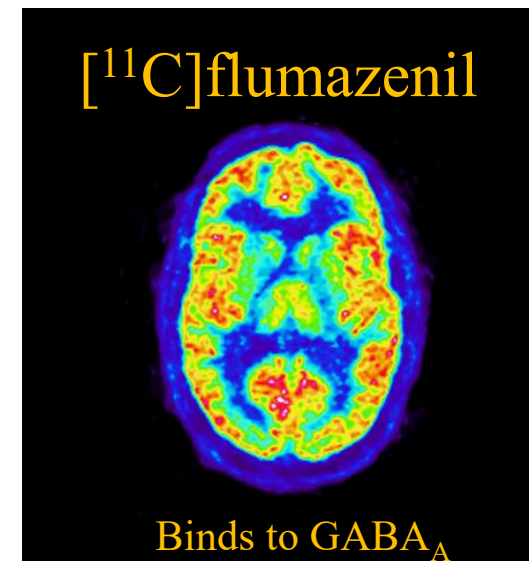
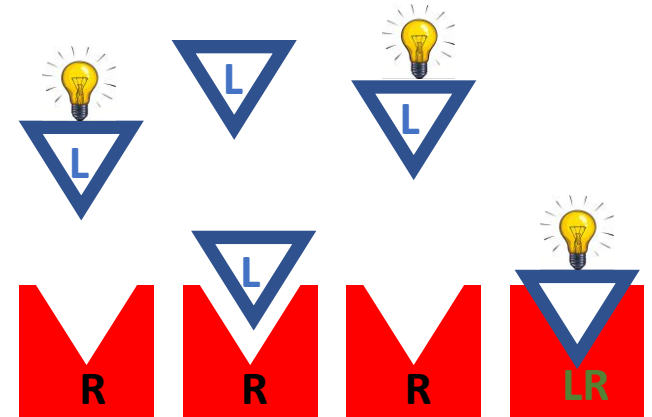
## Single binding site model

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Probably we want to estimate the number of receptors, [R]  
( $B_{\max}$  or  $B_{\text{avail}}$ )



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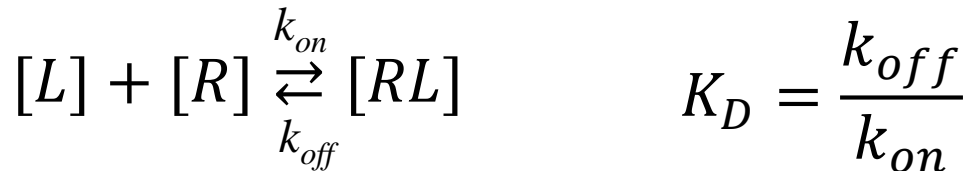


# Some useful concepts from biochemistry

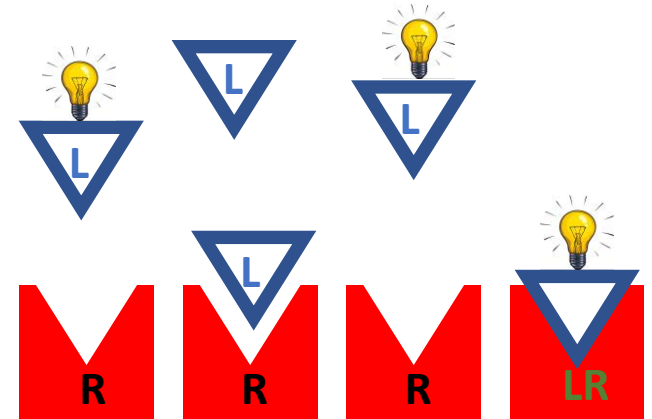
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Michelis-Menten equation

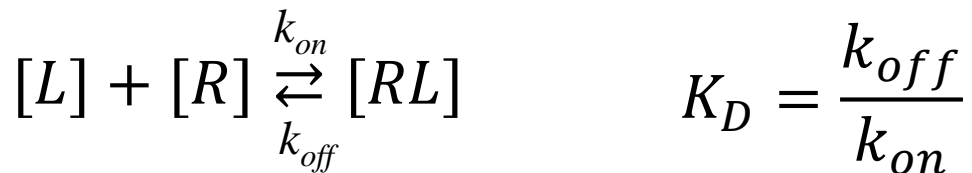
$$[RL] = \frac{[R][L]}{[L] + K_D}$$

# Some useful concepts from biochemistry

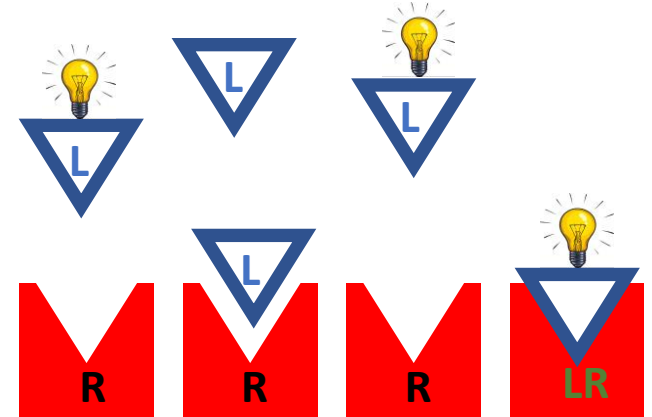
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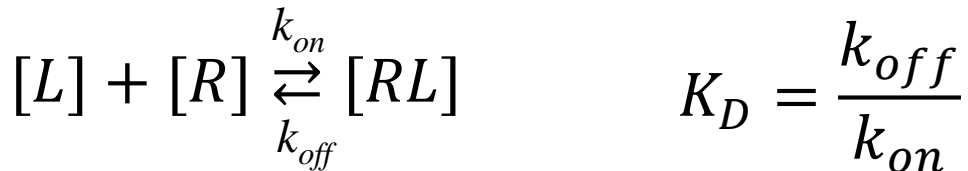
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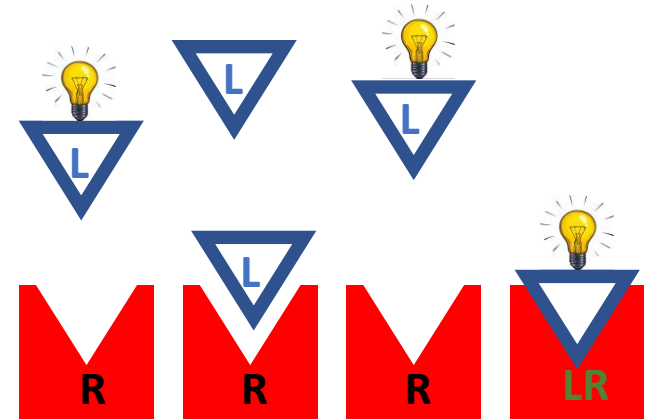
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Michelis-Menten equation

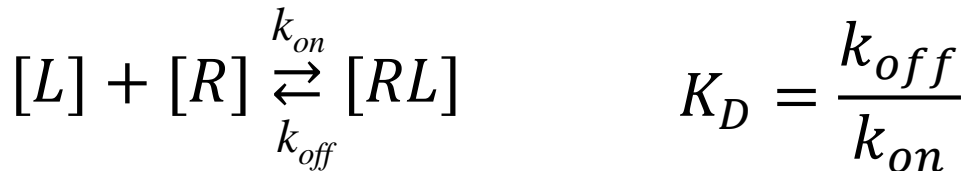
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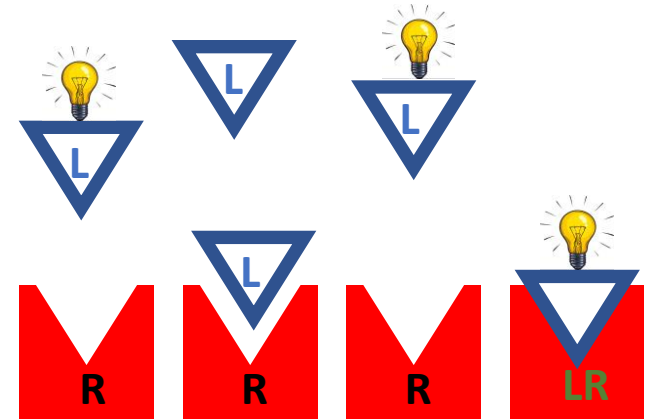
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Michelis-Menten equation

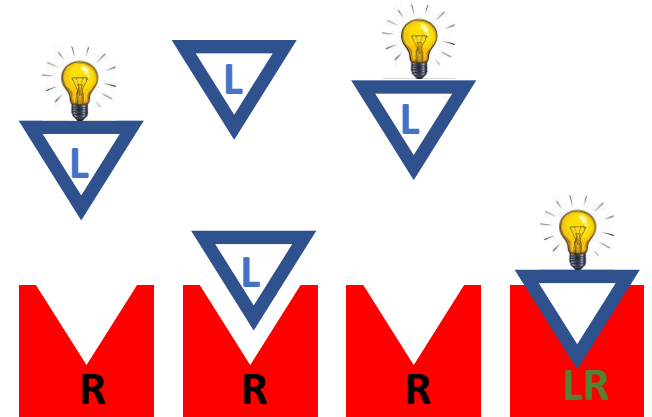
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# Some useful concepts from biochemistry

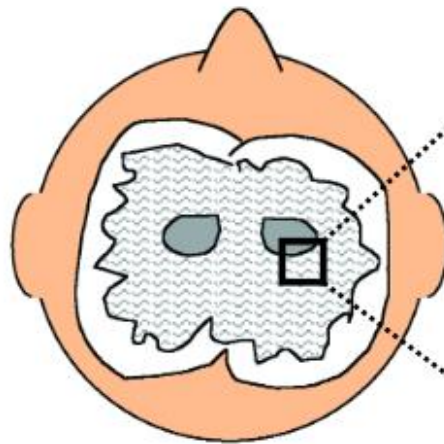
## Single binding site model

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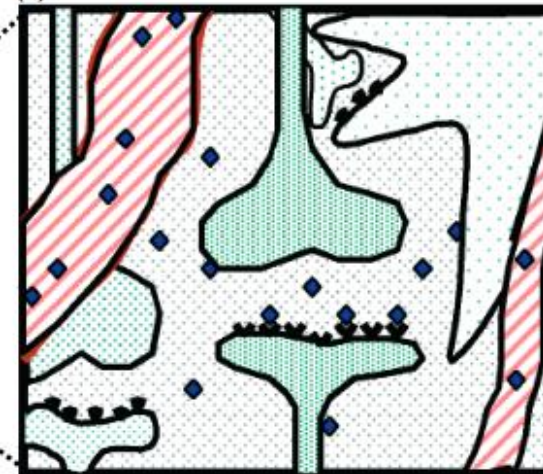
$$BP = \frac{\textit{bound}}{\textit{free}} = \dots = \frac{B_{max}}{K_D}$$



(a)



(b)



Morris et al., Emission Tomography, Elsevier 2004

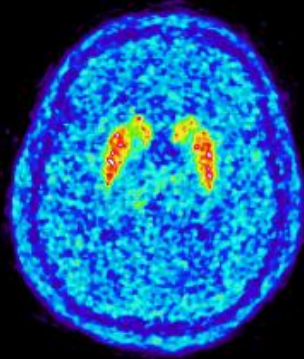
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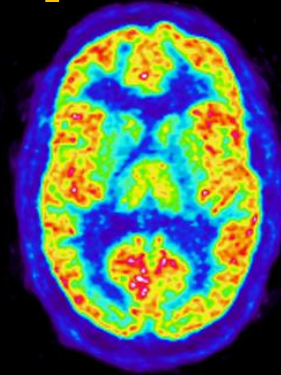


# PET/SPECT data quantification

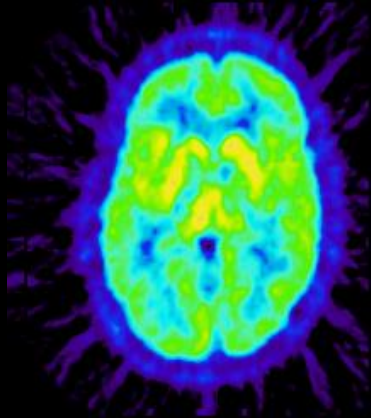
[<sup>11</sup>C]raclopride



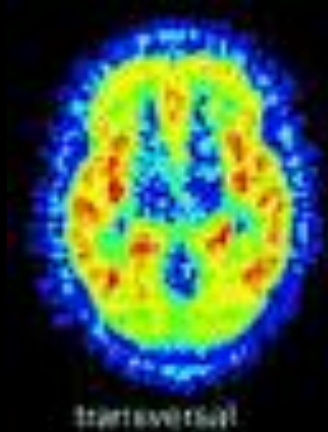
[<sup>11</sup>C]flumazenil



[<sup>11</sup>C]MADAM

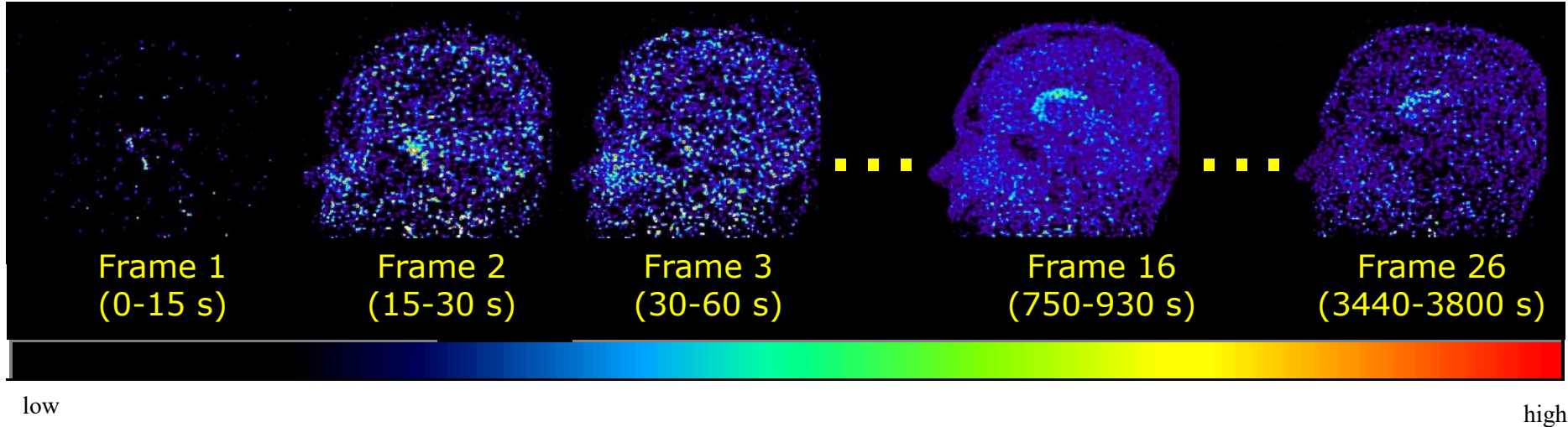


[<sup>11</sup>C]WAY-100635



# Time frames of a PET image

time →



Increasing frame durations (why?)

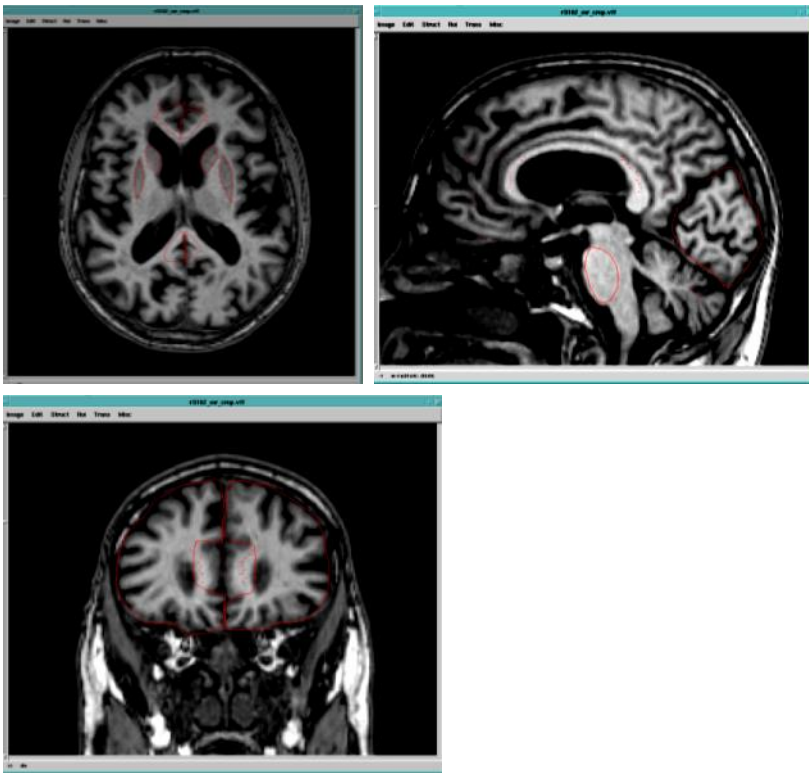
Each voxel (pixel) has a value, what's the unit?

Very noisy

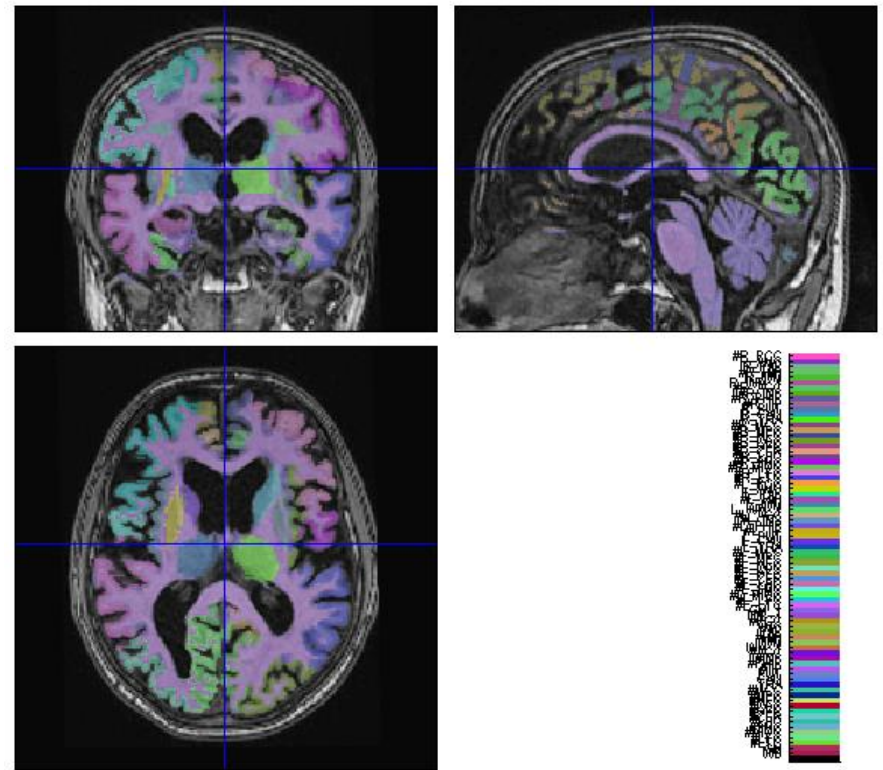
Very little spatial information

# Regions of Interests (ROI)

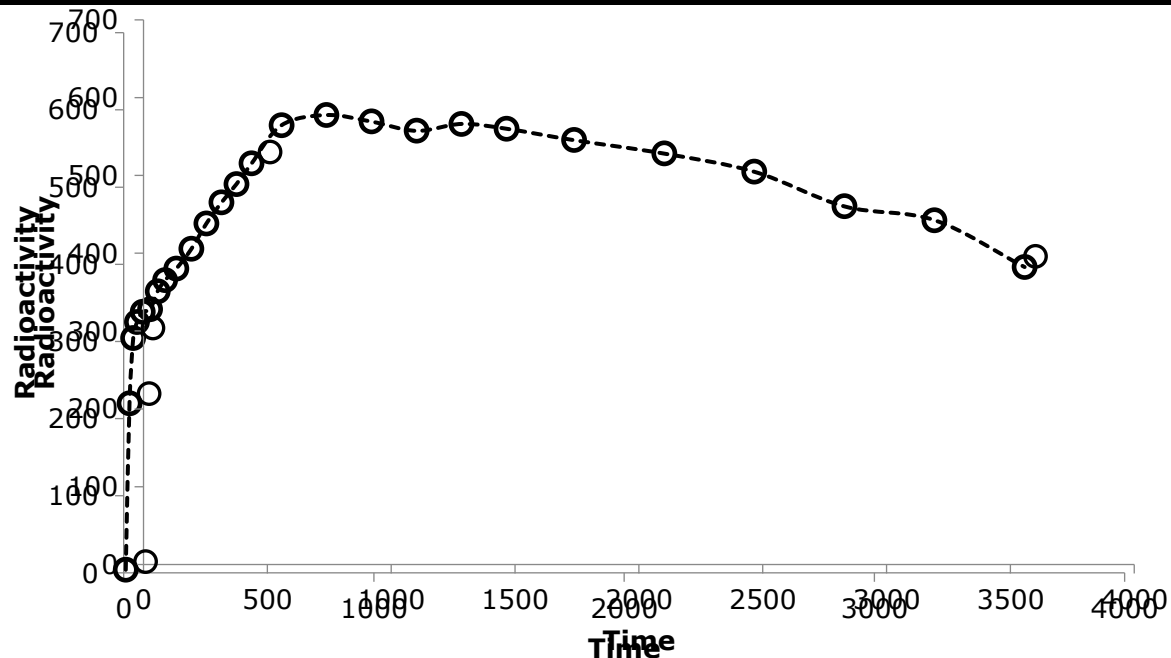
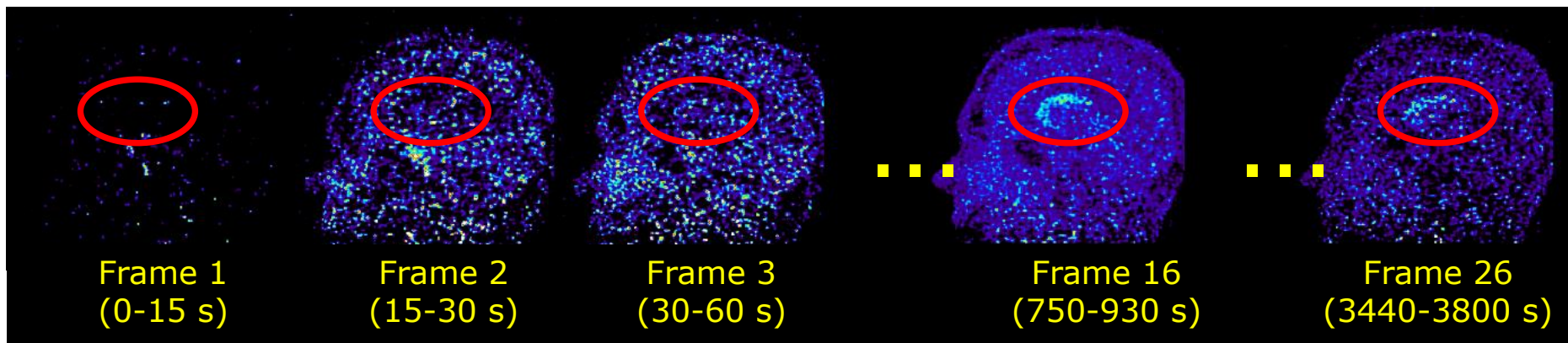
## Manual ROIs



## Automatic ROIs

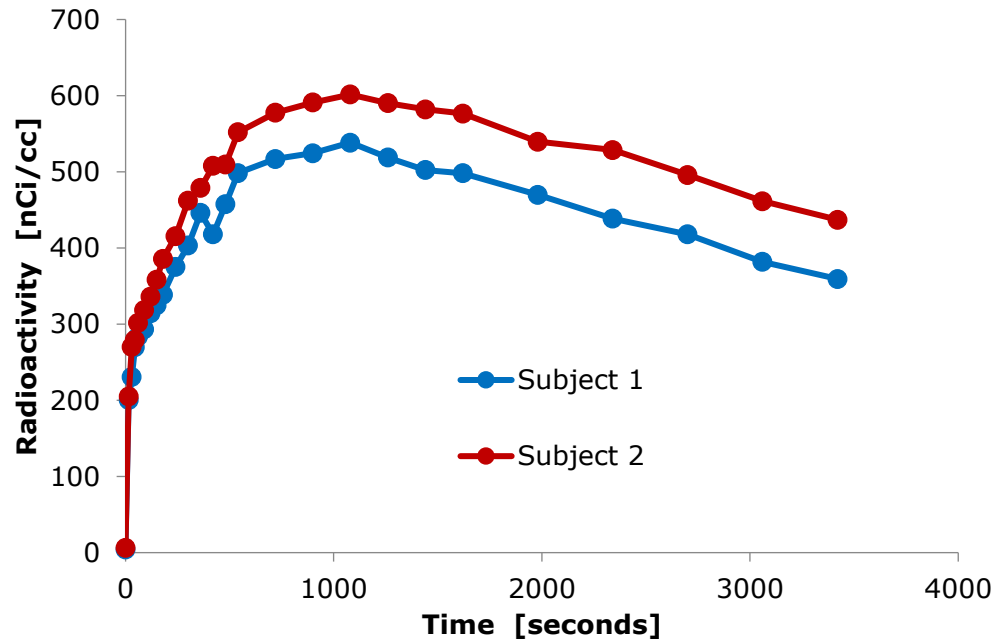


# Time frames of PET / SPECT images



Time activity curve (TAC)

# Quantification of dynamic PET / SPECT data



Subject 2 has a higher density of Dopamine D2/D3 receptors !?!

Was the same amount of radioactivity injected both subjects?

Did they have the same body weight?

Did the radioligand metabolise in the exact same way?

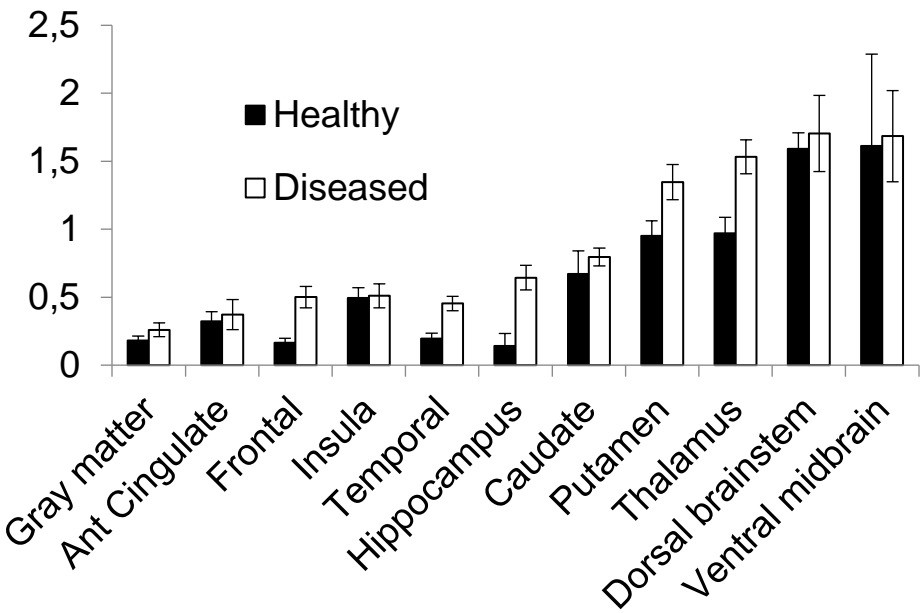
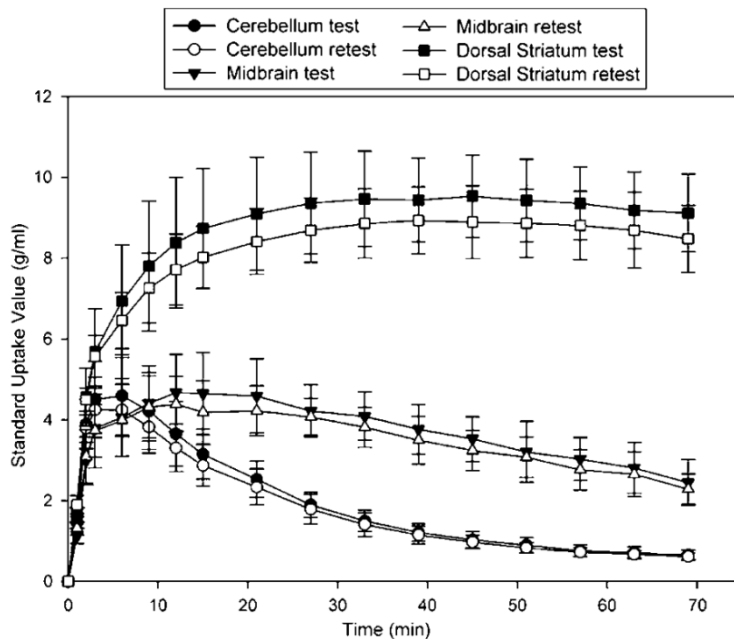
Did they have the same degree of non-specific binding?

Etc...

# Standardized uptake value

$$SUV(t) = \frac{TAC(t)}{ID / weight}$$

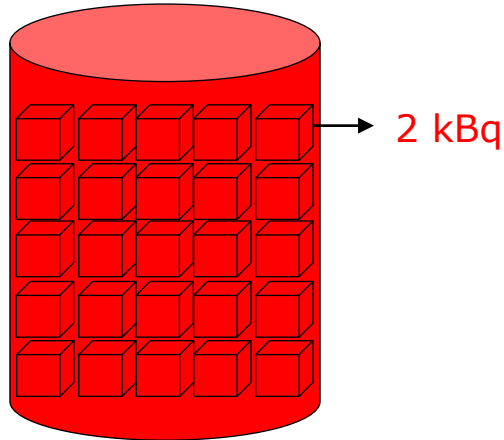
$$AUC_{SUV} = \int_0^{t_{end}} SUV(t) dt$$



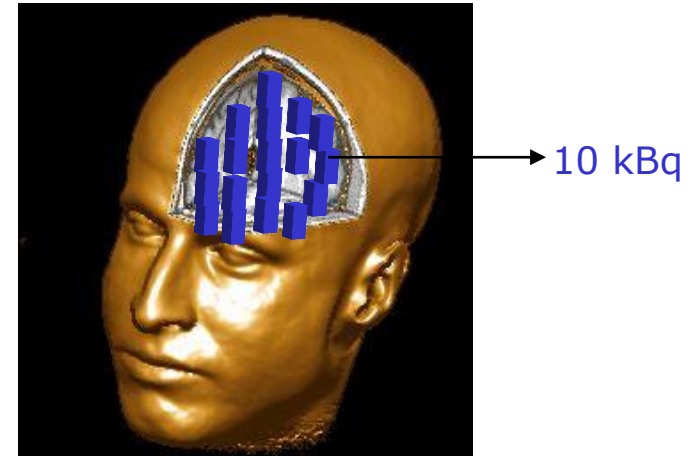
SUV curves, [<sup>11</sup>C]PE2I.  
Hirvonen et al., JCBFM 2008

# Total distribution volume, $V_T$

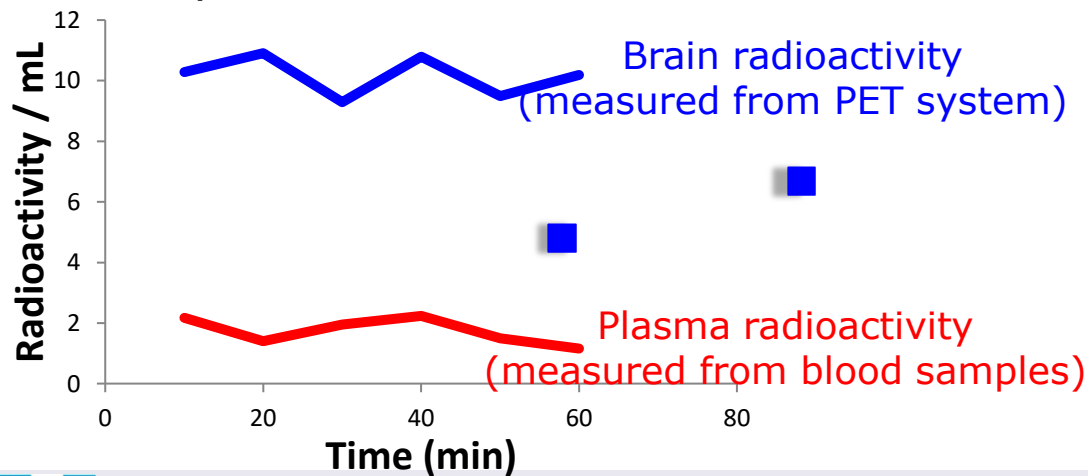
Parent radioligand in plasma



Radioactivity in brain



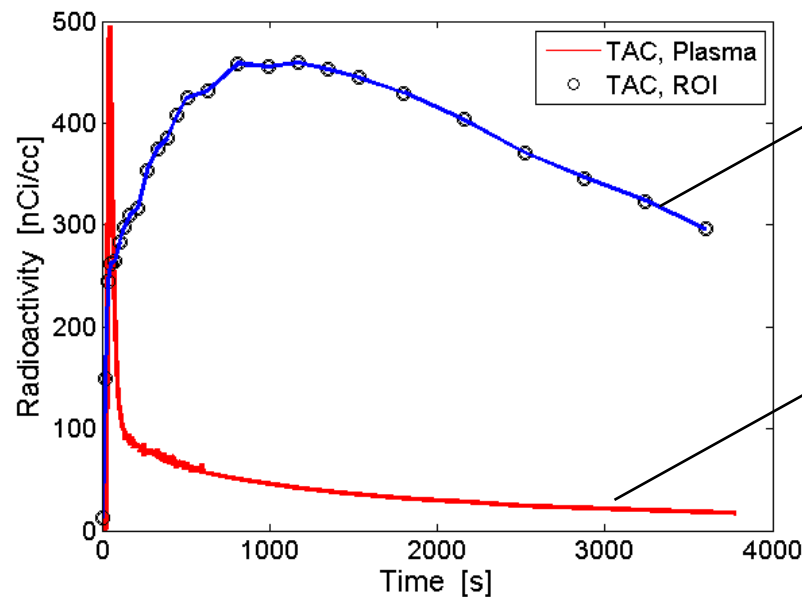
If equilibrium:



While at equilibrium,  
 $V_T = \text{brain} / \text{plasma}$

# Distribution volume, $V_T$

The volume of plasma required to account for the measured radioactivity in tissue.



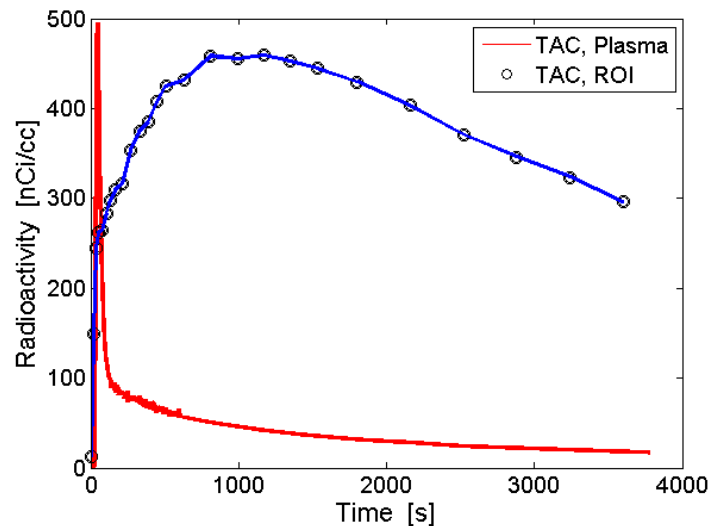
Concentration of radioligand in a brain region

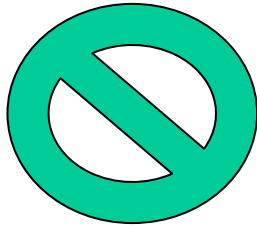
Concentration of unchanged radioligand in plasma



# Distribution volume, $V_T$

The volume of plasma required to account for the measured radioactivity in tissue.

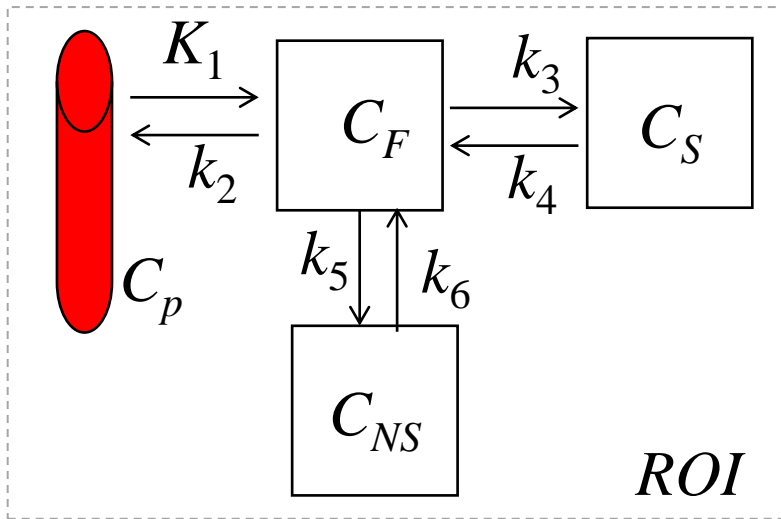


$$V_T = \frac{\int_0^{\infty} TAC(t) dt}{\int_0^{\infty} plasma(t) dt}$$


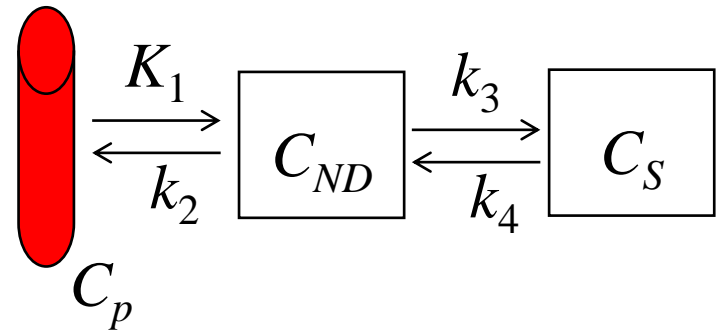
# Distribution volume, $V_T$

What can we assume about the kinetic behaviour of the tracer?

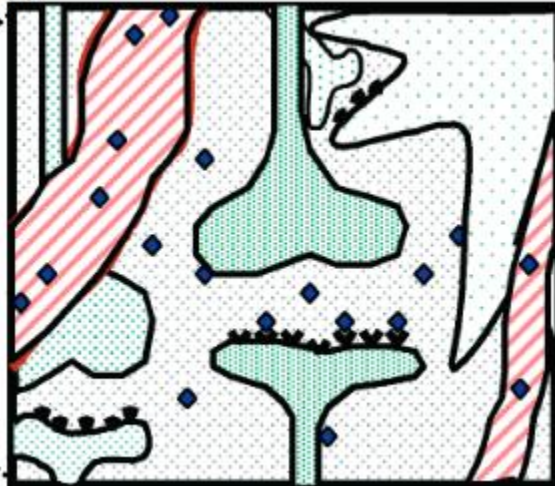
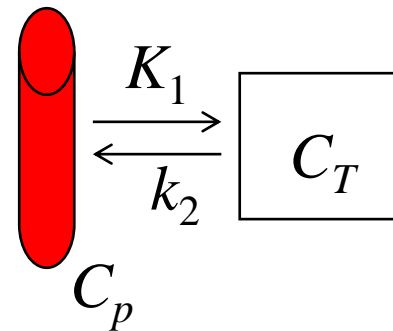
3 tissue compartment model



2 tissue compartment model

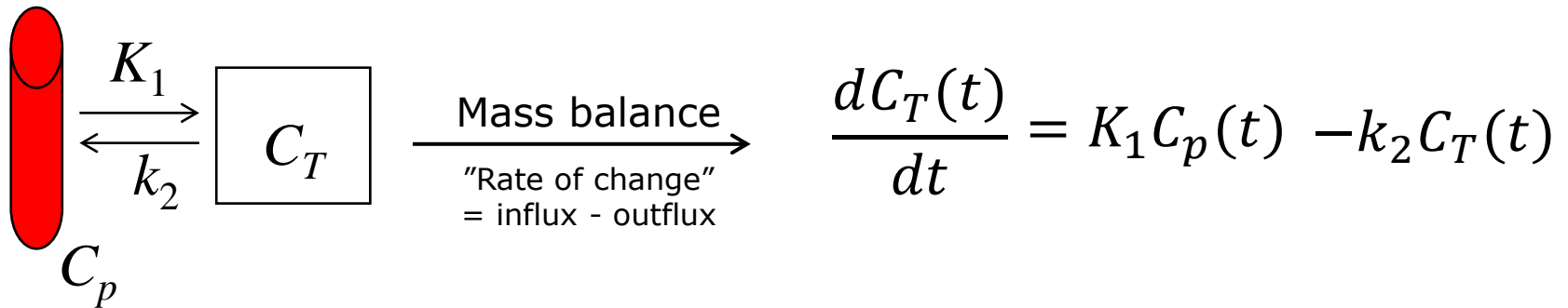


1 tissue compartment model



# Distribution volume, $V_T$

What can we assume about the kinetic behaviour of the tracer?



Solving dif.  
Eq... →

$$C_T(t) = C_p(t) \otimes K_1 e^{-k_2 t}$$

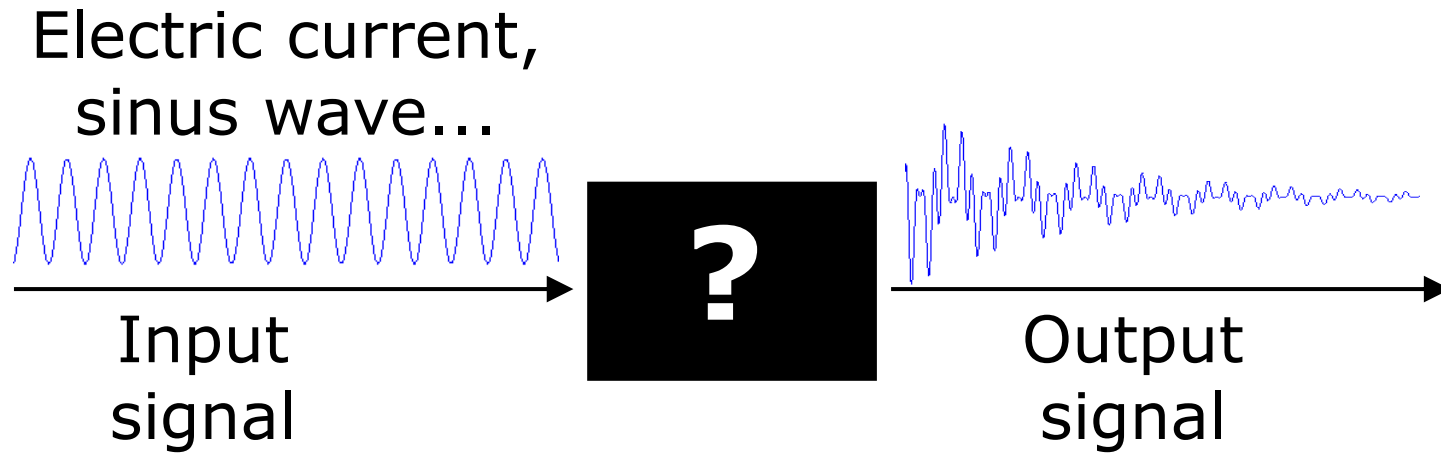
$$V_T = \frac{\int_0^{\infty} TAC(t) dt}{\int_0^{\infty} plasma(t) dt} = \frac{\int_0^{\infty} C_T(t) dt}{\int_0^{\infty} C_p(t) dt} = \dots = \frac{K_1}{k_2}$$

# Linear Time Invariant (LTI) Systems



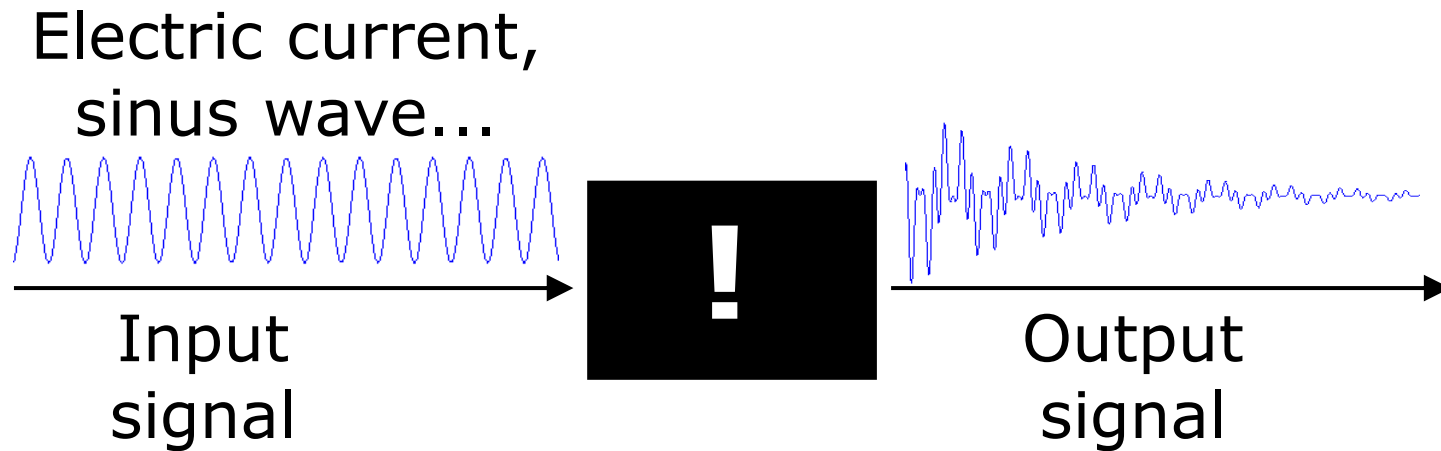
# Linear Time Invariant (LTI) Systems

## Synthesizer



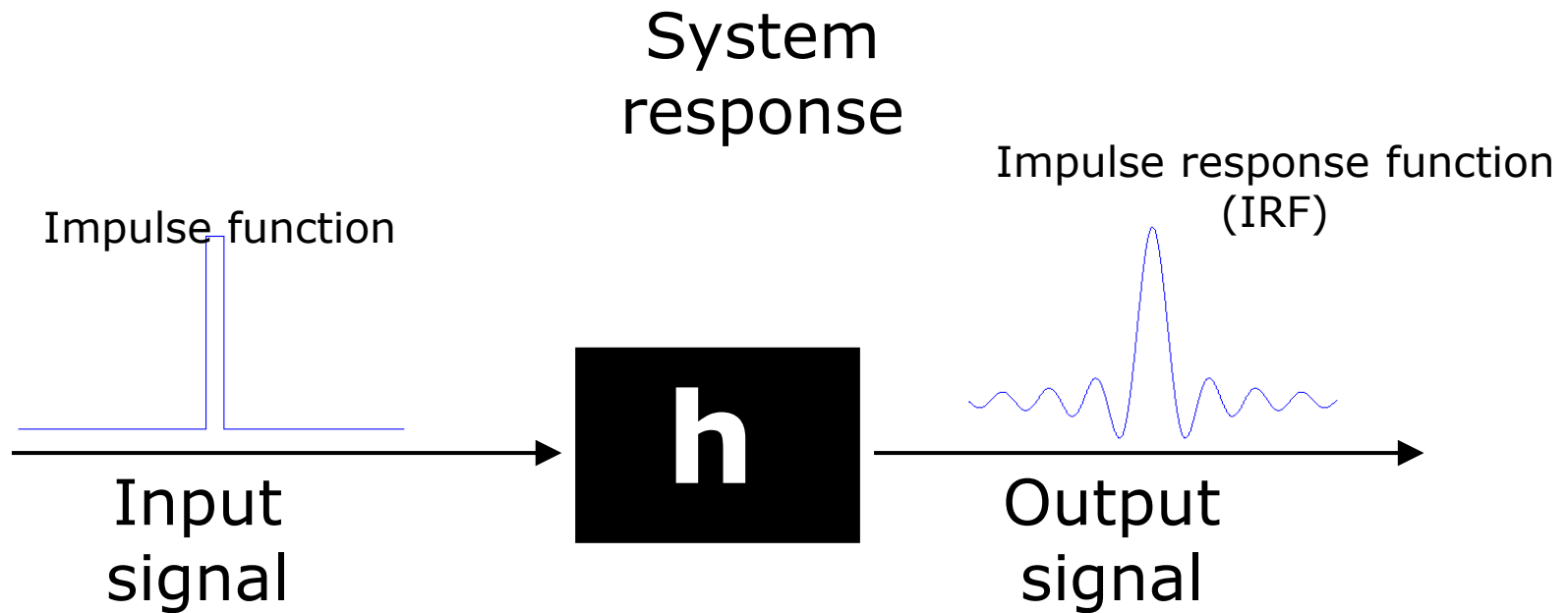
# Linear Time Invariant (LTI) Systems

## Synthesizer



Modify input signal + Study output signal →  
Understand the system (!)

# Linear Time Invariant (LTI) Systems



Impulse response function "defines" the system!

For any input signal  $f_i(t)$ , the corresponding output signal  $f_o(t)$  is given by

$$f_o(t) = f_i(t) \otimes IRF$$

# Convolution

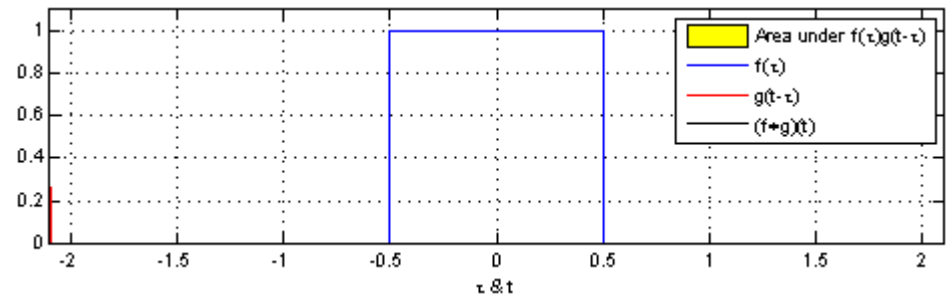
Convolution of two square functions. The convolution (black line) at any time is the size of the joint area (yellow field) of the two functions at that time.

Convolution of a square **input function** and a **"response function"**. The convolved signal will be the output in this **"model"**.



# Convolution

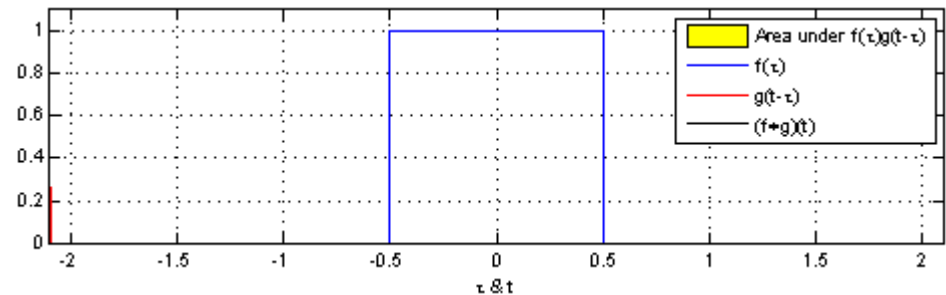
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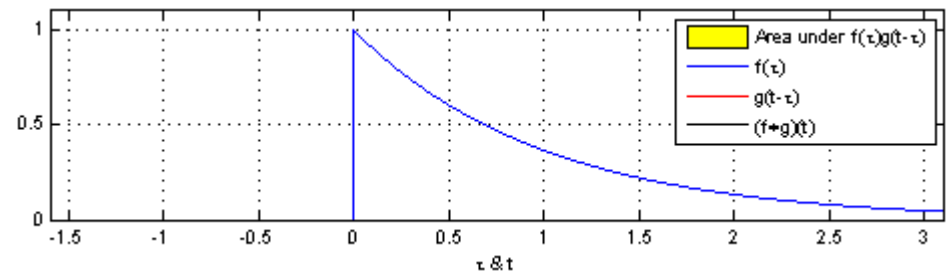
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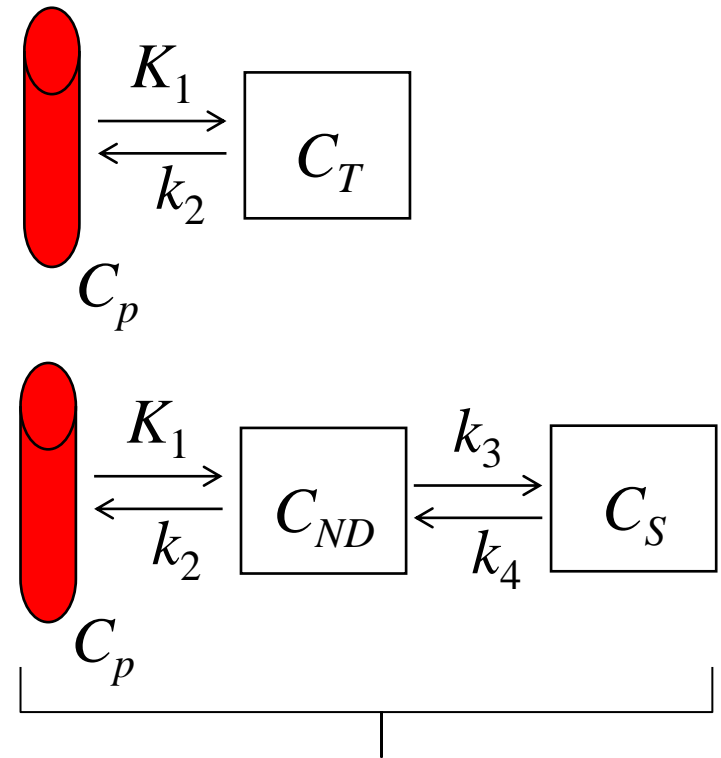
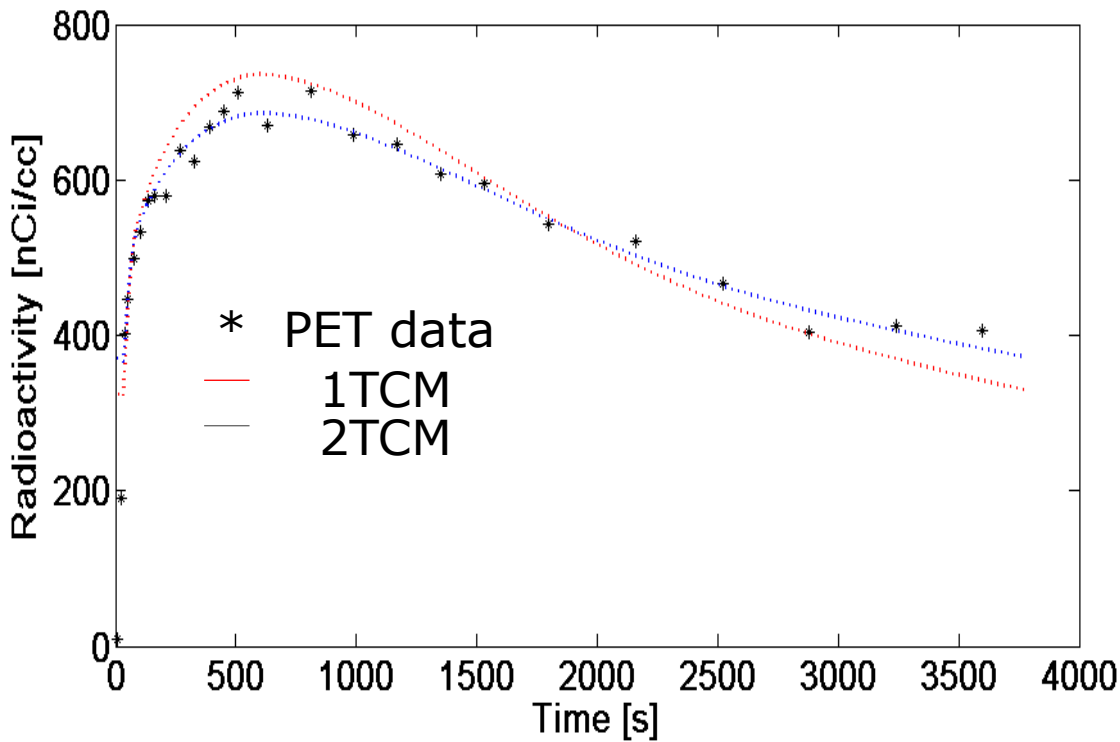
Convolution of a square **input function** and a **"response function"**. The convolved signal will be the output in this **"model"**.



# Distribution volume, $V_T$

$$C_T(t) = C_p(t) \otimes K_1 e^{-k_2 t}$$

$$V_T = \frac{\int_0^\infty C_T(t) dt}{\int_0^\infty C_p(t) dt} = \dots = \frac{K_1}{k_2}$$



$$V_T = \frac{K_1}{k_2} \left( 1 + \frac{k_3}{k_4} \right)$$

# Distribution volume, $V_T$

## Question?

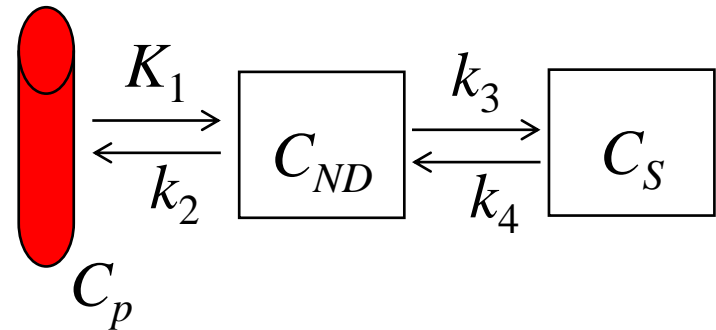
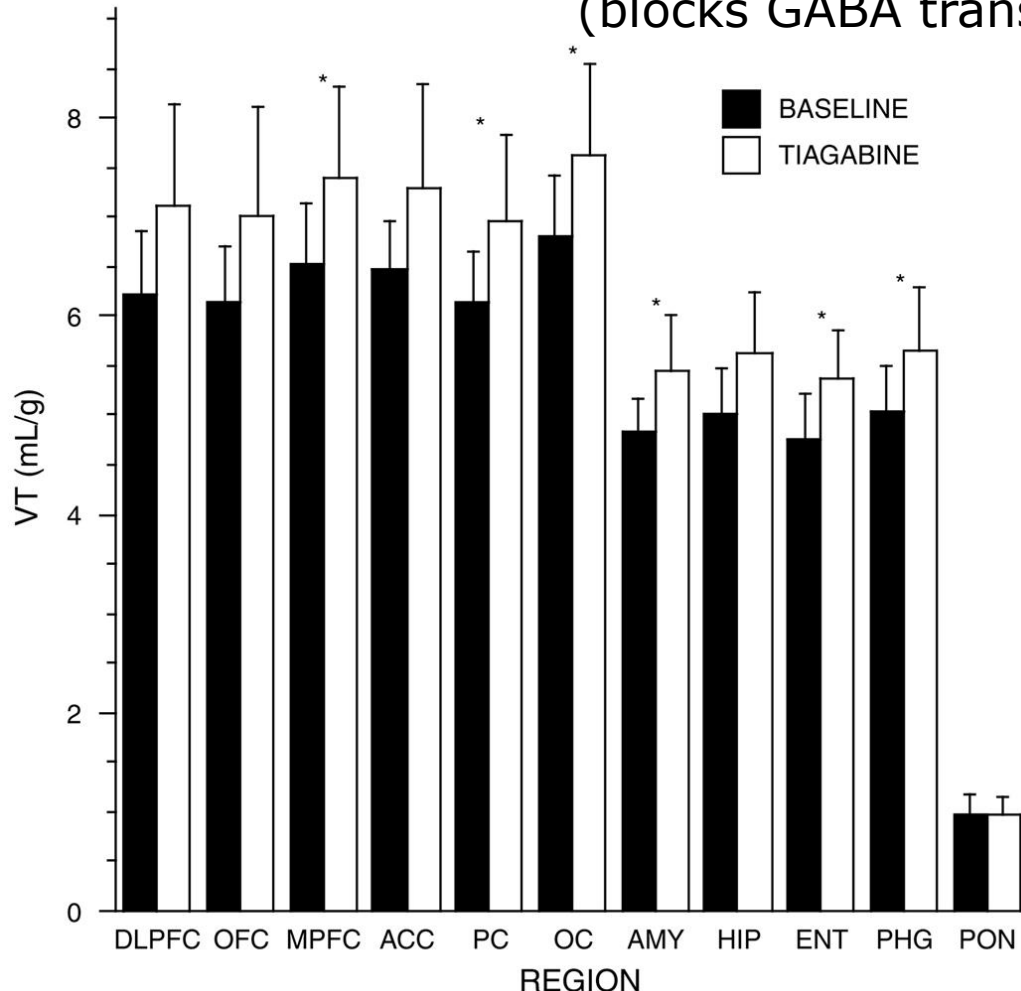
What if you think that the fits obtained from 1TCM and 2TCM are equally good?

Occam's razor  
Parsimony Principle

# Distribution volume, $V_T$

## Example

[ $^{11}\text{C}$ ]Flumazenil  $V_T$  before and after treatment with Tiagabine (blocks GABA transporter).



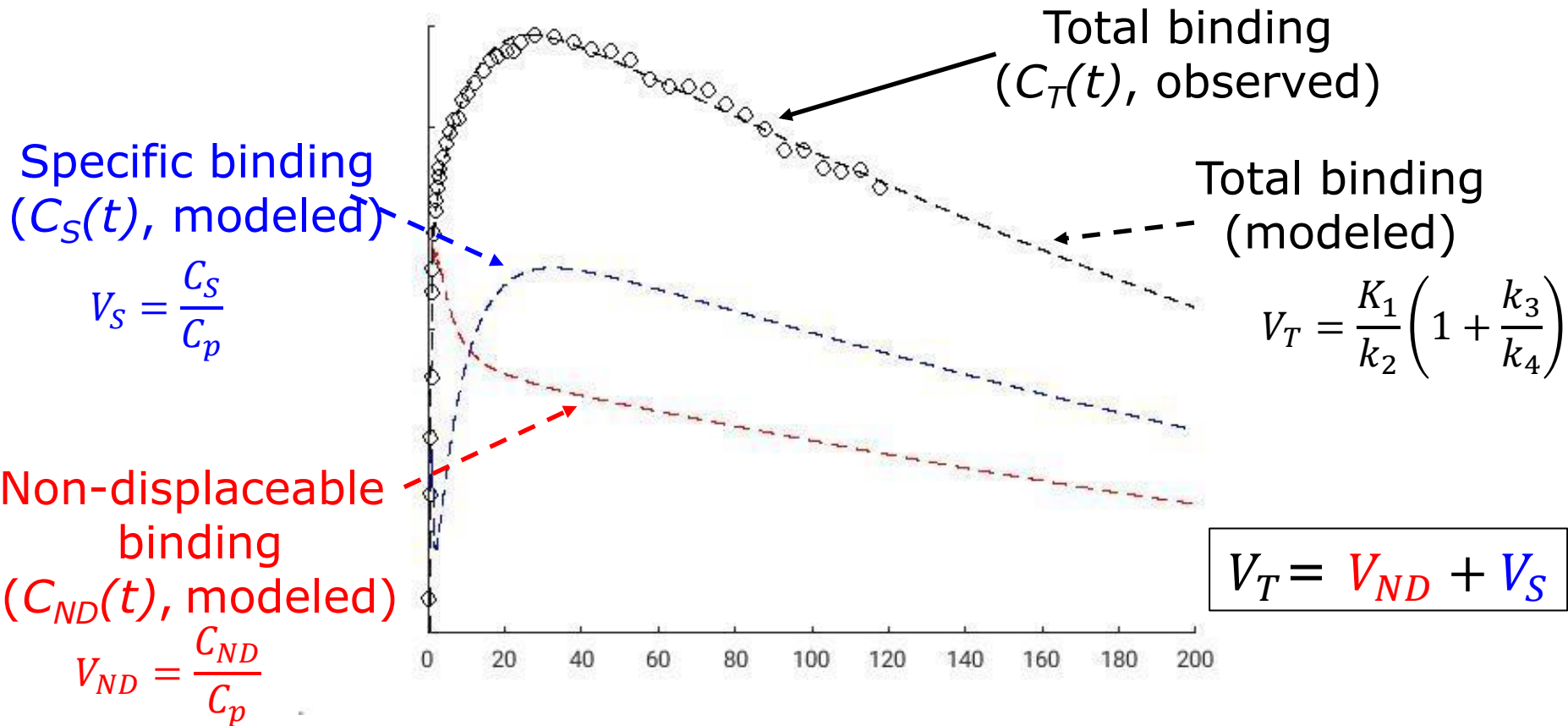
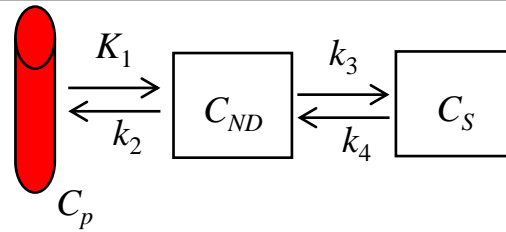
Frankle et al, *Neuropsychopharmacology* (2009)

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# Distribution volume, $V_T$

$$V_T = V_{ND} + V_S$$



# Distribution volume, $V_T$

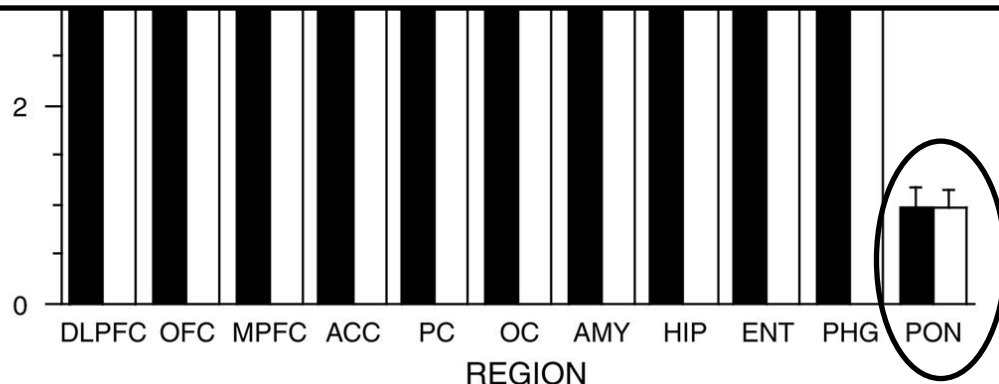
## Example

[ $^{11}\text{C}$ ]Flumazenil  $V_T$  before and after treatment with Tiagabine (blocks GABA transporter).

The results of this study are consistent with the hypothesis that the acute increases in extracellular cortical GABA can be detected as an increase in the binding of the BDZ site-specific radiotracer, [ $^{11}\text{C}$ ]flumazenil. The principle underlying this hypothesis is the 'GABA shift'—the enhancement in BDZ-receptor affinity for BDZ site substrates resulting from the increased GABA (Tallman *et al*, 1978; Braestrup *et al*, 1982). It is widely accepted that

$V_T$  (mL/g)

Derivation of BDZ parameters was based upon the following assumptions: (1) because of the low density of BDZ in the pons (Abadie *et al*, 1992; Price *et al*, 1993), pons  $V_T$  was assumed to be representative of equilibrium nonspecific binding,  $V_{ND}$ ; (2) the nonspecific binding did not vary significantly between regions.



$C_p$

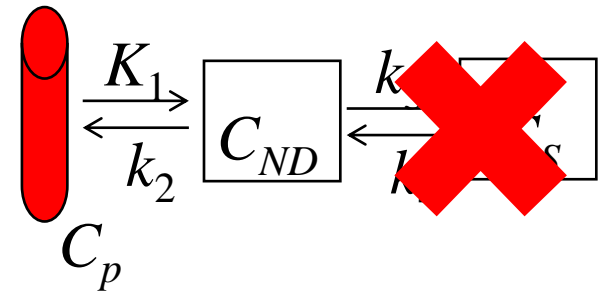
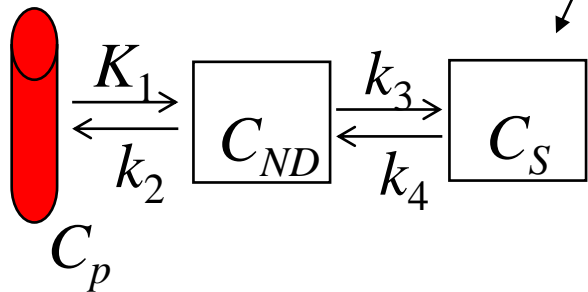
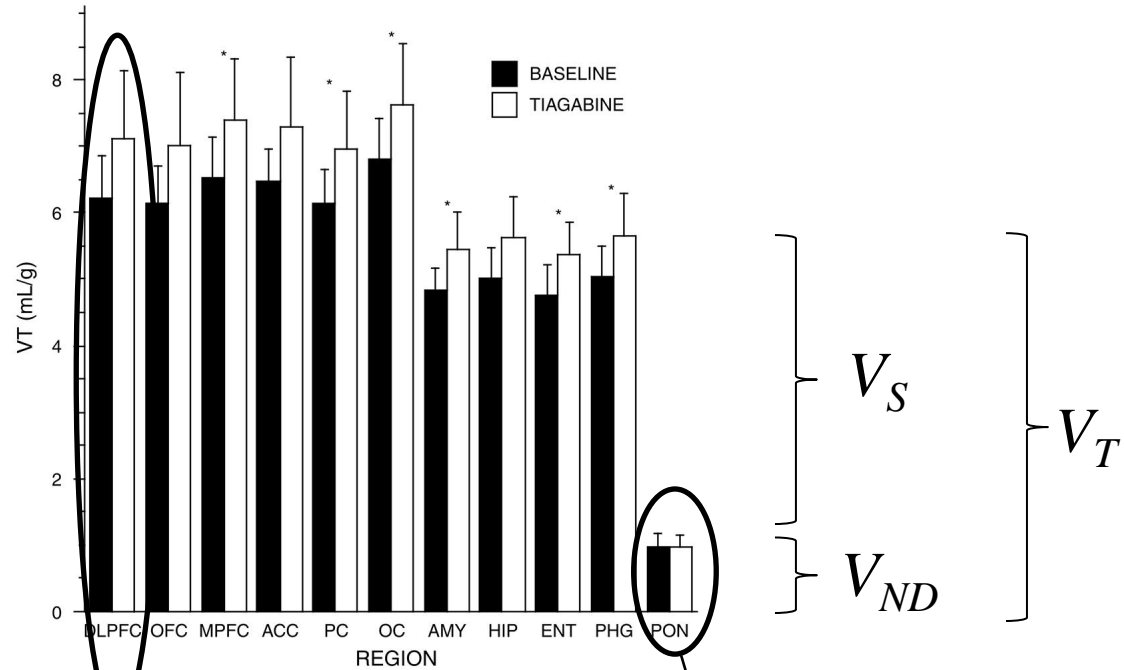
Reference region – negligible density of target  
 $C_S=0$

Frankle *et al*, *Neuropsychopharmacology* (2009)

# Fundamental assumption of PET:

$V_{ND}$  doesn't change

$$V_T = V_S + V_{ND}$$





# How can we estimate $V_{ND}$ ?

**If there is a reference region:**

Your answer here...

**If there is not a reference region:**

Your answer here...

# How can we estimate $V_{ND}$ ?

## **If there is a reference region:**

Apply your kinetic model to the reference region TAC.  
The  $V_T$  you get will be  $V_{ND}$

## **If there is not a reference region:**

Your answer here...

# How can we estimate $V_{ND}$ ?

## **If there is a reference region:**

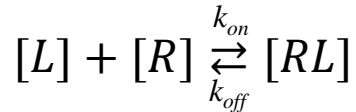
Apply your kinetic model to the reference region TAC.  
The  $V_T$  you get will be  $V_{ND}$

## **If there is not a reference region:**

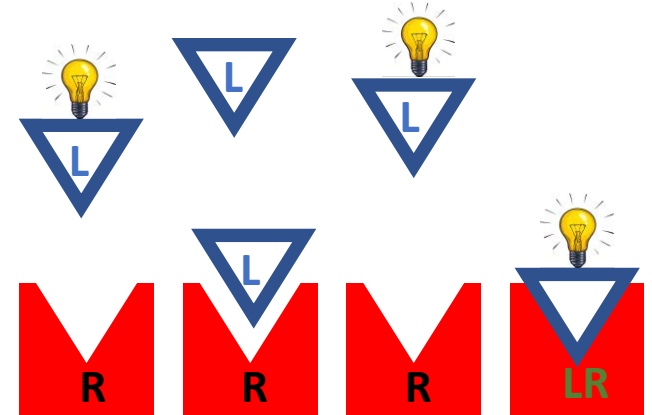
No established way exists...

# Some useful concepts from biochemistry

## Single binding site model (equilibrium)



$$K_D = \frac{k_{off}}{k_{on}} \quad BP = \frac{[RL]}{[L]} = \frac{B_{max}}{K_D}$$

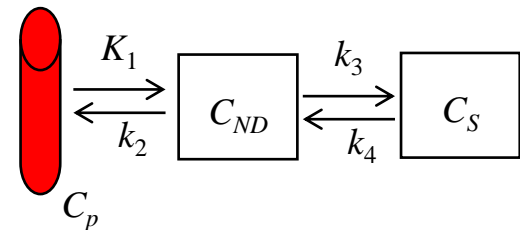


$f_{ND}$ : fraction of free tracer in ND compartment  
 $f_p$ : fraction of free tracer in plasma

$$[RL]: C_S$$

$$[L]: f_{ND} C_{ND}$$

$$[L]: f_p C_p$$



# Three approaches to estimate BP

## Approach 1 (relative to ND)

$$BP = \frac{[RL]}{[L]} = \frac{B_{max}}{K_D}$$

$$\begin{aligned} [RL]: C_S \\ [L]: f_{ND} C_{ND} \\ [L]: f_p C_p \end{aligned}$$

$f_{ND}$ : fraction of free tracer in ND compartment  
 $f_p$ : fraction of free tracer in plasma  
 $C_p$ : Concentration of tracer in plasma

### Approach 1: "Free" means "free in non-displaceable compartment"

$$\frac{[RL]}{[L]} = \frac{C_S}{f_{ND} C_{ND}} = \frac{C_S / C_p}{f_{ND} C_{ND} / C_p} = \frac{V_S}{f_{ND} V_{ND}} \rightarrow \frac{V_T - V_{ND}}{V_{ND}} = f_{ND} \frac{B_{max}}{K_D} = BP_{ND}$$

# Three approaches to estimate BP

## Approach 2 (relative to concentration in plasma)

$$BP = \frac{[RL]}{[L]} = \frac{B_{max}}{K_D}$$

$$\begin{aligned} [RL]: C_S \\ [L]: f_{ND} C_{ND} \\ [L]: f_p C_p \end{aligned}$$

$f_{ND}$ : fraction of free tracer in ND compartment  
 $f_p$ : fraction of free tracer in plasma  
 $C_p$ : Concentration of tracer in plasma

### Approach 2: "Free" means "free in plasma"

(Conc. of free in tissue = conc. of free in plasma – but  $f_p$  doesn't change across groups)

$$\frac{[RL]}{[L]} = \frac{C_S}{f_p C_p} = \frac{C_S / C_p}{f_p C_p / C_p} \rightarrow V_T - V_{ND} = f_p \frac{B_{max}}{K_D} = BP_P$$

# Three approaches to estimate BP

## Approach 3 (relative to free conc in plasma)

$$BP = \frac{[RL]}{[L]} = \frac{B_{max}}{K_D}$$

$$\begin{aligned} [RL]: C_S \\ [L]: f_{ND} C_{ND} \\ [L]: f_p C_p \end{aligned}$$

$f_{ND}$ : fraction of free tracer in ND compartment  
 $f_p$ : fraction of free tracer in plasma  
 $C_p$ : Concentration of tracer in plasma

### Approach 3: "Free" means "free in plasma"

(i.e., conc. of free in plasma = conc. of free in tissue)

$$\frac{[RL]}{[L]} = \frac{C_S}{f_p C_p} = \frac{C_S / C_p}{f_p} = \frac{V_S}{f_p} = \frac{V_T - V_{ND}}{f_p} = \frac{B_{max}}{K_D} = BP_F$$

# Binding potential!

## PET

$B_{max}$ : Total number of receptors

$K_D$ : Affinity of the radioligand

$f_p$ : Free fraction of radioligand in plasma

$f_{ND}$ : Free fraction of radioligand in non-displaceable compartment

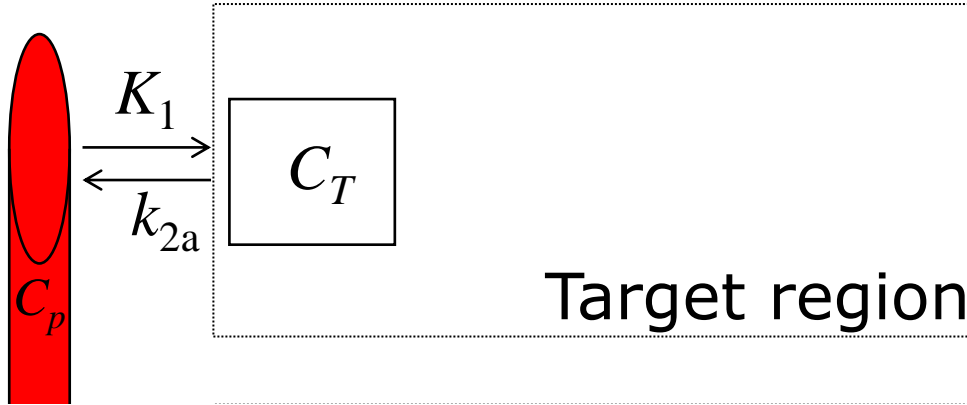
$$BP = \frac{B_{max}}{K_D}$$

Diagram illustrating the derivation of three forms of Binding Potential (BP) from the equation  $BP = \frac{B_{max}}{K_D}$ :

- Top branch:  $\frac{B_{max}}{K_D} = \frac{V_T - V_{ND}}{f_p} = BP_F$
- Middle branch:  $\frac{B_{max}}{K_D} f_p = V_T - V_{ND} = BP_P$
- Bottom branch:  $\frac{B_{max}}{K_D} f_{ND} = \frac{V_T - V_{ND}}{V_{ND}} = BP_{ND}$



# Simplified Reference Tissue Model



$$C_T(t) = C_p(t) \otimes K_1 e^{-k_2 t}$$

# Summary of (some) assumptions

## If you use 2TCM to estimate $V_T$ :

- Assume that the model describes the data well
- Assume that  $V_{ND}$  is not different between the groups

## If you use 2TCM to estimate $BP_{ND} (V_T - V_{ND}) / V_{ND}$ :

- Assume that the model describes the data well
- Assume that  $V_{ND}$  is the same in all brain regions
- Assume that  $V_{ND}$  is not different between the groups

## If you use reference tissue modeling to estimate $BP_{ND}$

- Assume that the model describes the data well
- Assume that  $V_{ND}$  is the same in all brain regions
- Assume that  $V_{ND}$  is not different between the groups

# Summary of (some) assumptions

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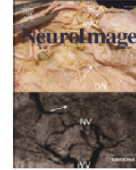
# Possible to estimate $V_{ND}$ without reference region?



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## Estimation of in vivo nonspecific binding in positron emission tomography studies without requiring a reference region



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<sup>b</sup> Department of Molecular Imaging and Neuropathology, New York State Psychiatric Institute, 10032, New York, NY, USA

<sup>c</sup> Departments of Psychiatry and Radiology, Stony Brook University, Stony Brook, 11794, NY, USA



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## Accuracy and reliability of [<sup>11</sup>C]PBR28 specific binding estimated without the use of a reference region

Pontus Plaven-Sigra<sup>a,\*</sup>, Martin Schain<sup>b</sup>, Francesca Zanderigo<sup>b,c</sup>, Karolinska [<sup>11</sup>C]PBR28 study group<sup>d</sup>, Ilan Rabiner<sup>d</sup>, Roger Gunn<sup>d,e</sup>, Todd Ogden<sup>b,c,f</sup>, Simon Cervenka<sup>a</sup>

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<sup>b</sup> Department of Psychiatry, Columbia University, New York, NY, USA

<sup>c</sup> Molecular Imaging and Neuropathology Division, New York State Psychiatric Institute, New York, USA

<sup>d</sup> Invivo LLC, London, UK

<sup>e</sup> Division of Brain Sciences, Imperial College London, London, UK

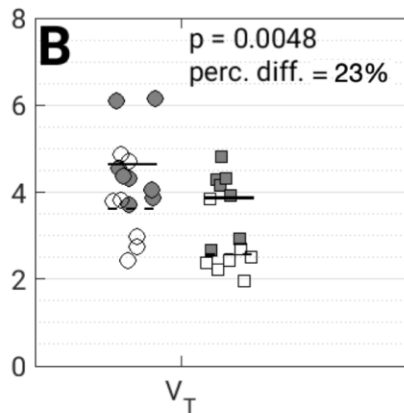
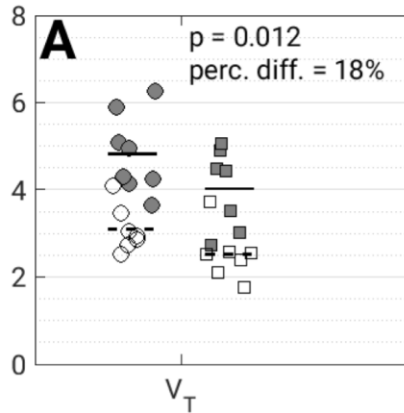
<sup>f</sup> Department of Biostatistics, Mailman School of Public Health, Columbia University, New York, USA

**Martin Schain**

NRU, Copenhagen University Hospital, Rigshospitalet



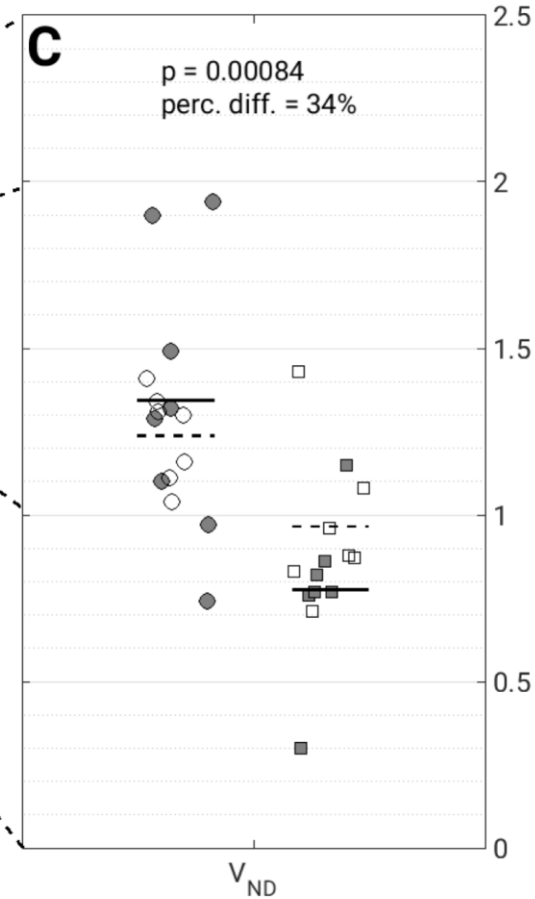
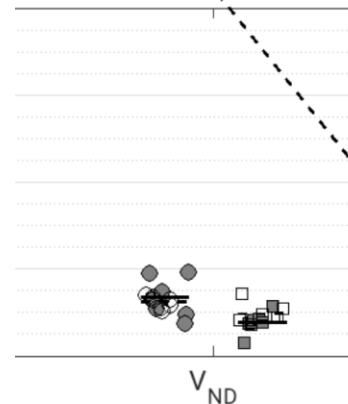
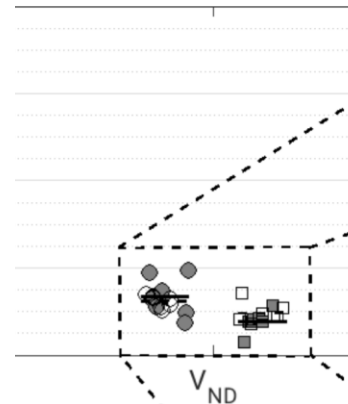
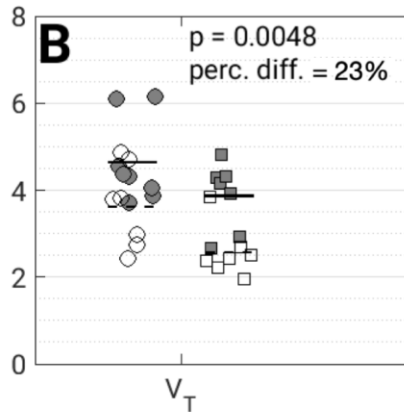
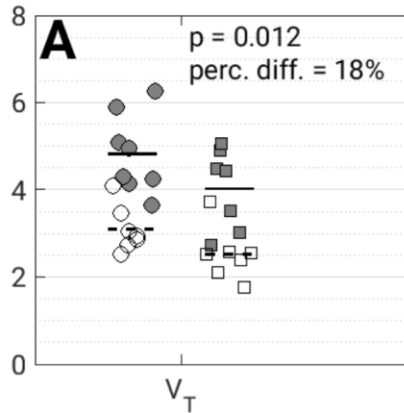
# Example where $V_{ND}$ may confound



● Ctrl (HAB) ○ Ctrl (MAB) ■ AUD (HAB) □ AUD (MAB) — mean (HAB) - - - mean (MAB)

Laurell et al., in Review

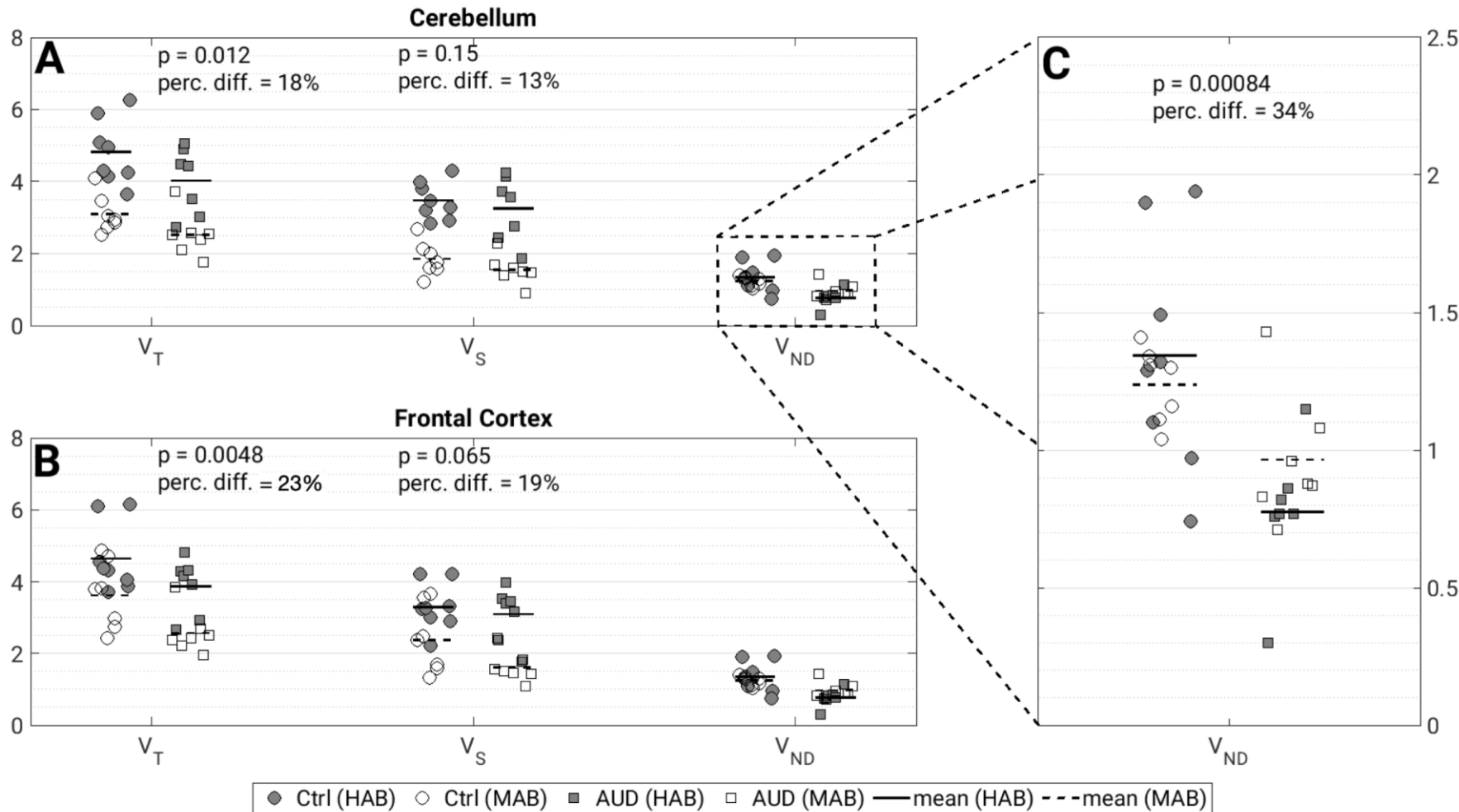
# Example where $V_{ND}$ may confound



● Ctrl (HAB) ○ Ctrl (MAB) ■ AUD (HAB) □ AUD (MAB) — mean (HAB) - - - mean (MAB)

Laurell et al., in Review

# Example where $V_{ND}$ may confound



Laurell et al., in Review

# Summary

- Dynamic PET data = Acquired over time → time activity curves
- Most (not all) radioligands can be described by 1TCM or 2TCM
- 1TCM and 2TCM requires arterial input functions (cumbersome measurement)
- With a TCM, non-linear regression is used to estimate rate constants, which are combined into total distribution volume ( $V_T$ )
- If a reference region exist, non-displaceable distribution volume ( $V_{ND}$ ) can be estimated → estimation of BP
- $BP_{ND}$ ,  $BP_F$  and  $BP_p$  are thought to represent estimates of  $B_{max}$ . This relies on a number of assumptions.