

Receptor kinetics

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Martin Schain
MSc, PhD

Antaros Medical
Bioventure Hub, Mölndal, Sweden
Mindpark, Lund, Sweden

Neurobiology Research Unit (NRU)
Copenhagen University Hospital



Overview

- What goes on in the blood?
- What goes on in the brain?
 - Some useful concepts from biochemistry
- Kinetic modeling of PET / SPECT data
- Compare outputs from our kinetic models to in vitro analyses
- Emphasize on some assumptions – please don't violate.

Established nomenclature for PET / SPECT

Journal of Cerebral Blood Flow & Metabolism (2007) 27, 1533–1539

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www.jcbfm.com



Review Article

Consensus nomenclature for *in vivo* imaging of reversibly binding radioligands

Robert B Innis¹, Vincent J Cunningham², Jacques Delforge³, Masahiro Fujita¹, Albert Gjedde⁴, Roger N Gunn⁵, James Holden⁶, Sylvain Houle⁷, Sung-Cheng Huang⁸, Masanori Ichise⁹, Hidehiro Iida¹⁰, Hiroshi Ito¹¹, Yuichi Kimura¹², Robert A Koeppe¹³, Gitte M Knudsen¹⁴, Juhani Knuuti¹⁵, Adriaan A Lammertsma¹⁶, Marc Laruelle², Jean Logan¹⁷, Ralph Paul Maguire¹⁸, Mark A Mintun¹⁹, Evan D Morris²⁰, Ramin Parsey⁹, Julie C Price²¹, Mark Slifstein⁹, Vesna Sossi²², Tetsuya Suhara¹¹, John R Votaw²³, Dean F Wong²⁴ and Richard E Carson²⁵



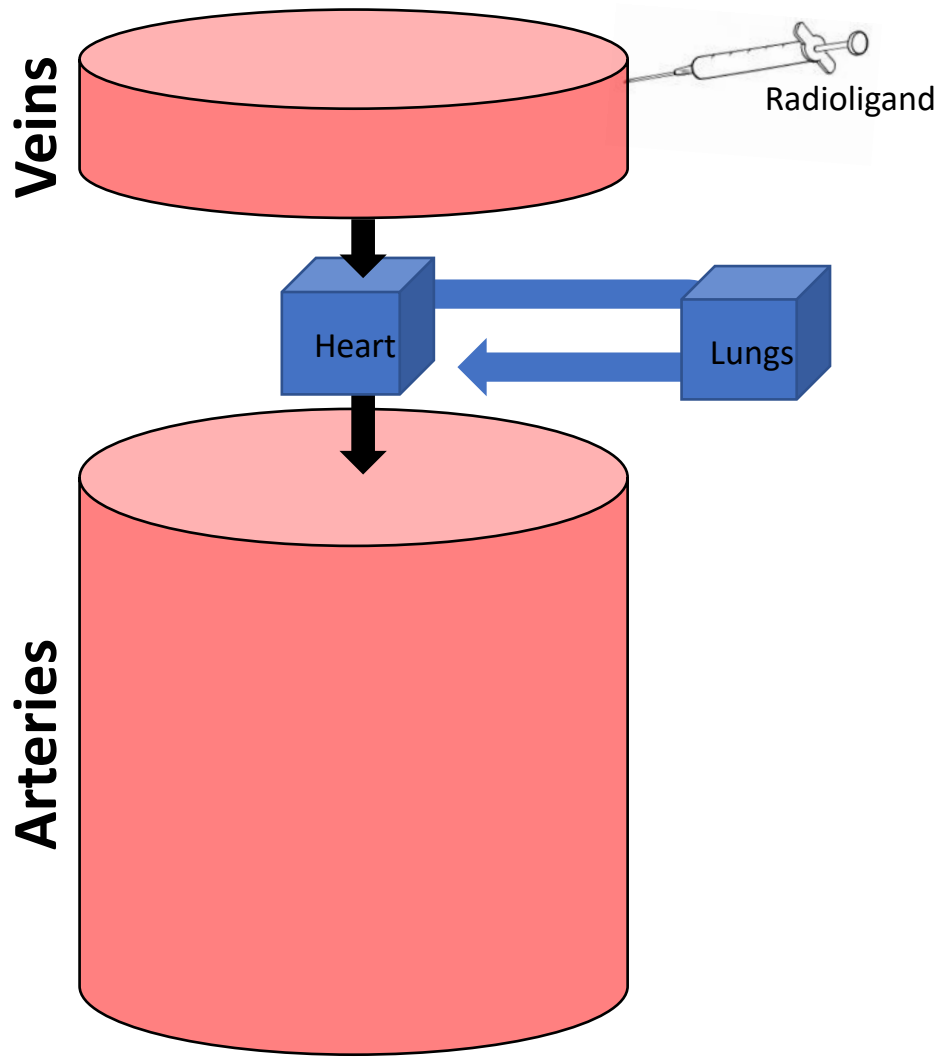
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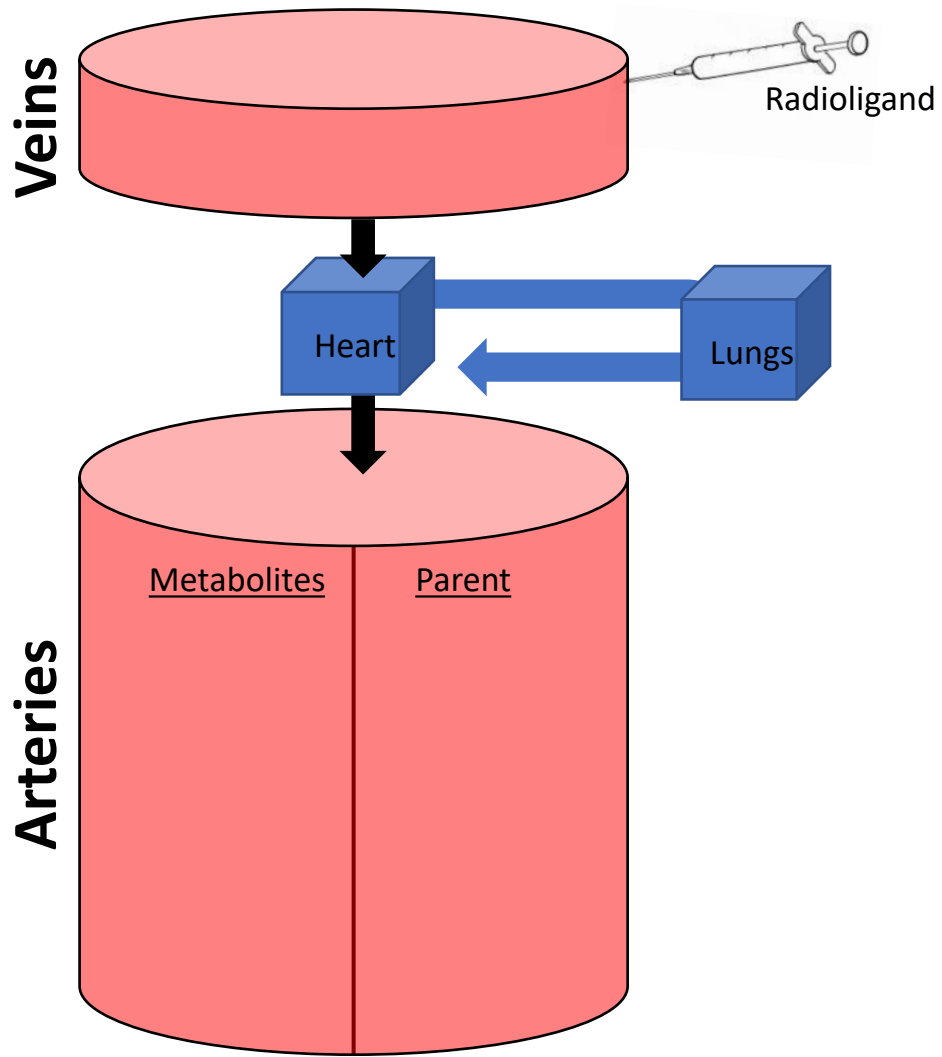
NRU, Copenhagen University Hospital, Rigshospitalet

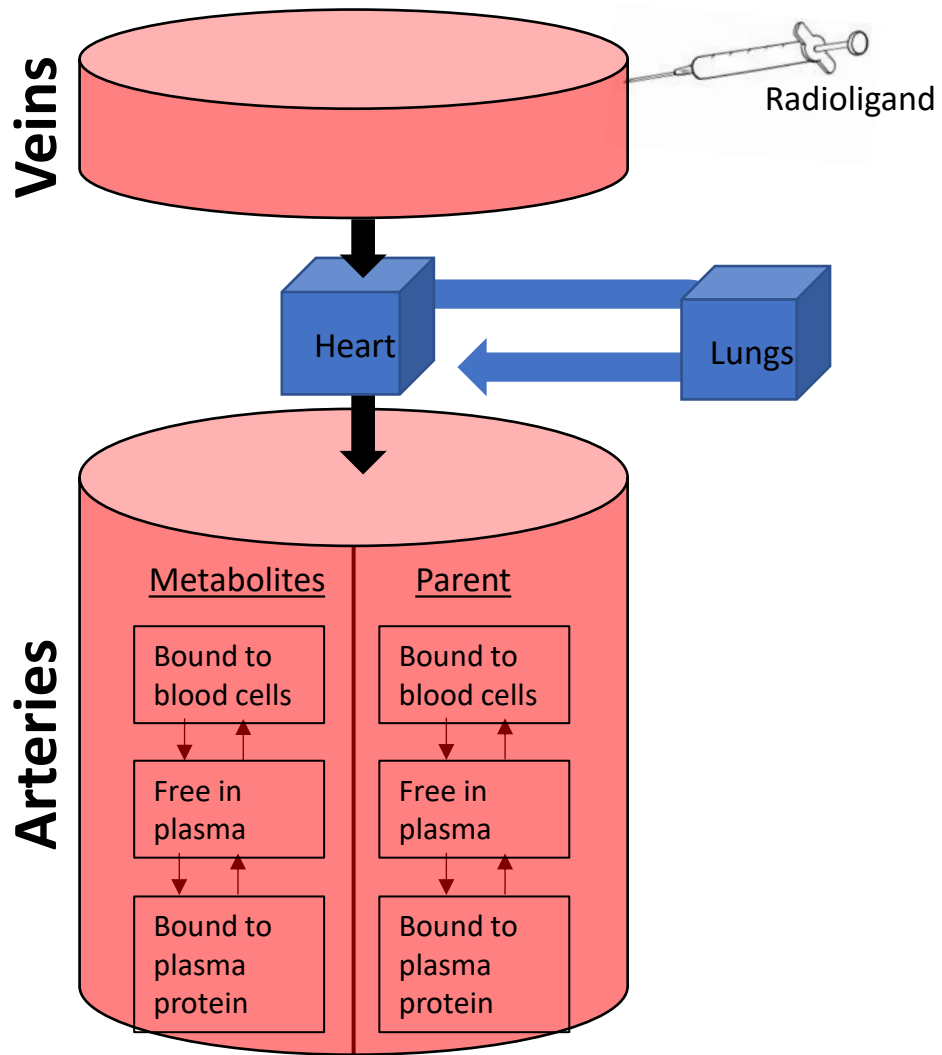


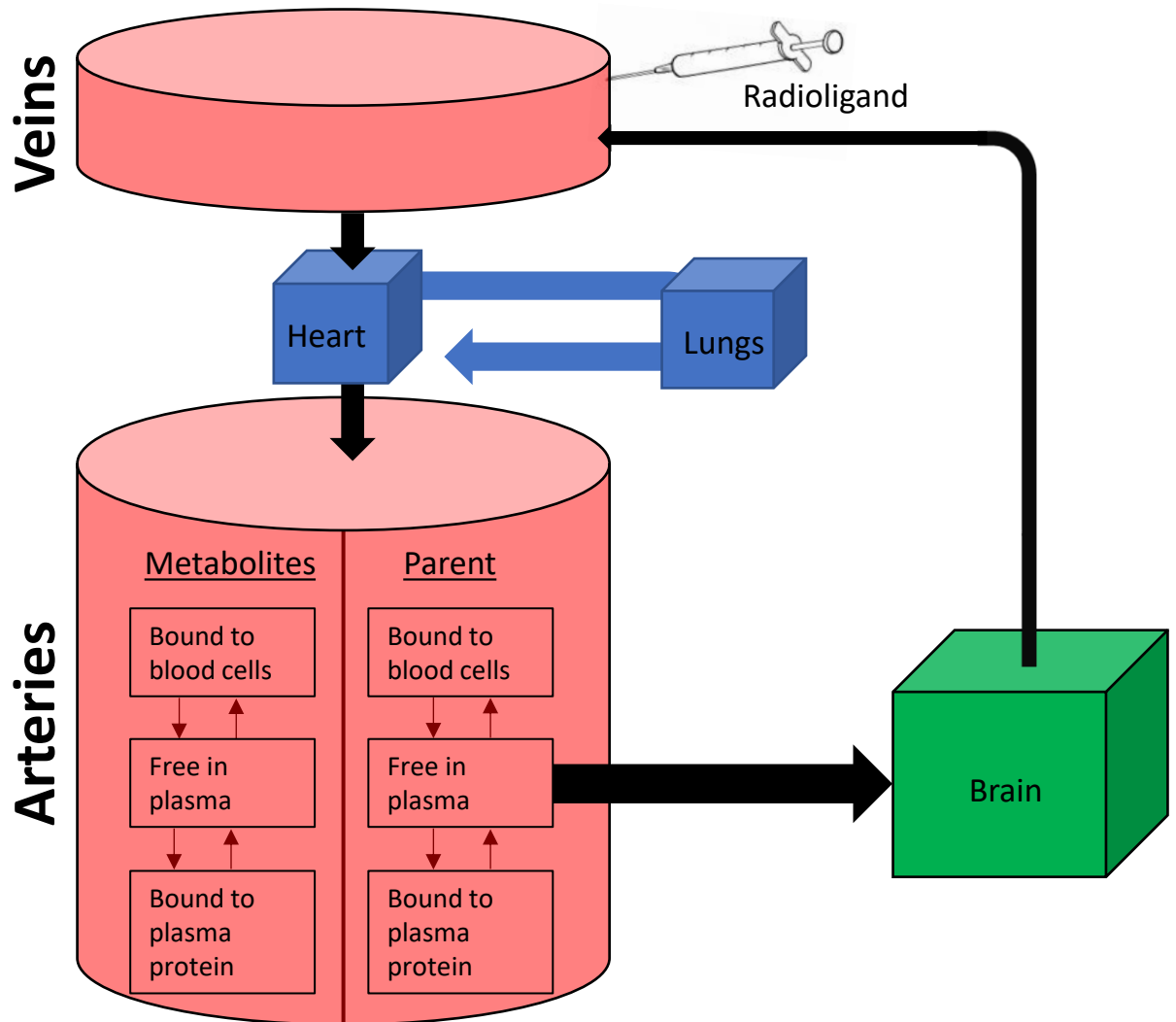
Antaros
Medical

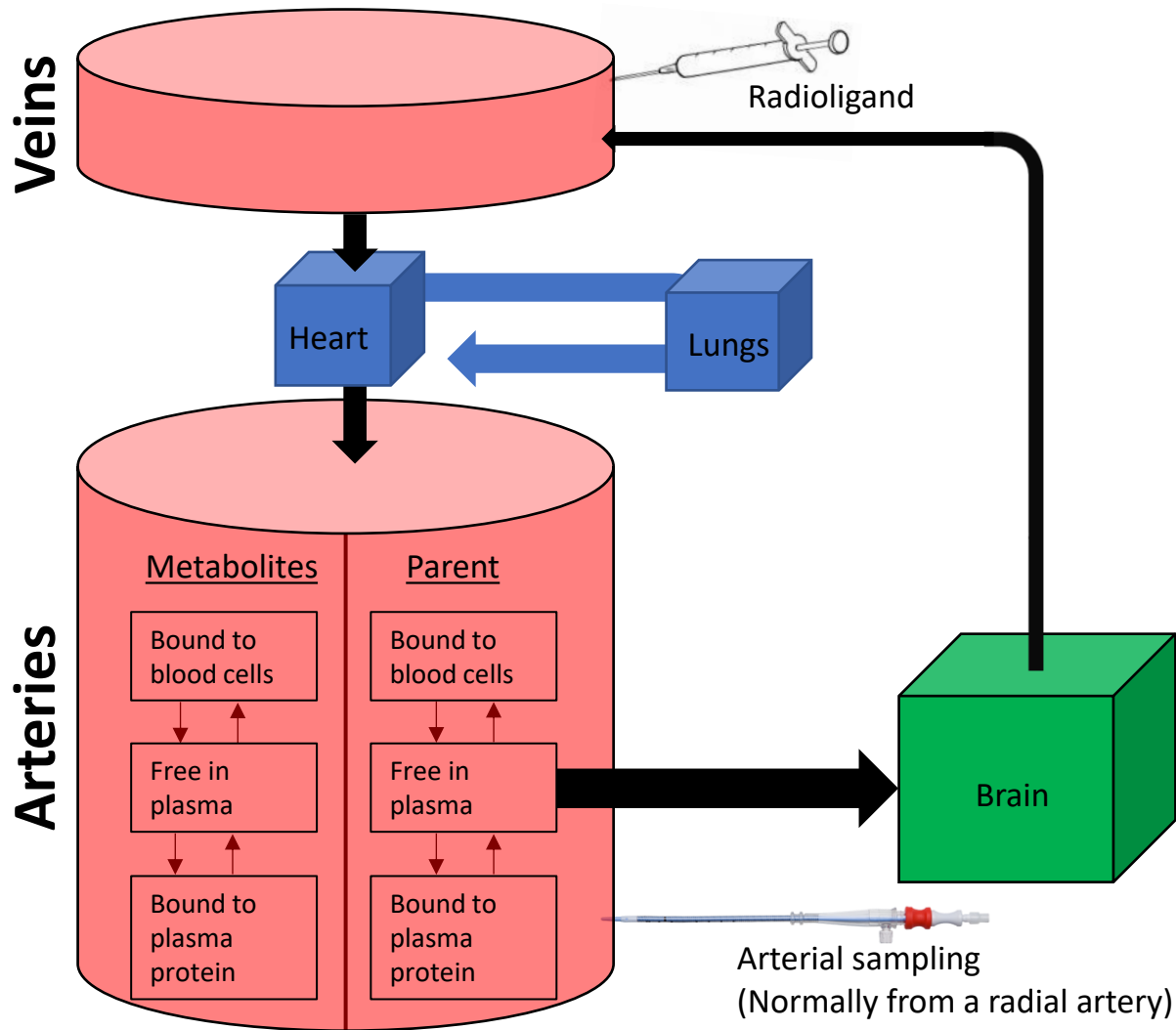










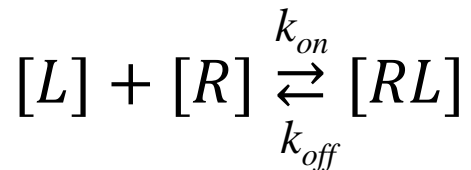


Some useful concepts from biochemistry

Single binding site model

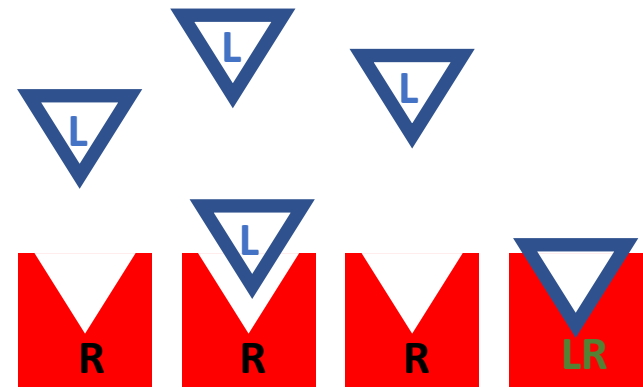
[RL]: Conc. of bound receptor-ligand complexes

[L] : Conc. of free ligand



Dissociation constant

$$K_D = \frac{k_{off}}{k_{on}} \quad \text{Affinity} = 1/K_D$$

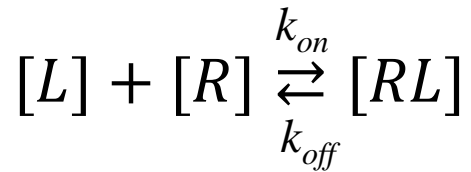


Binding potential

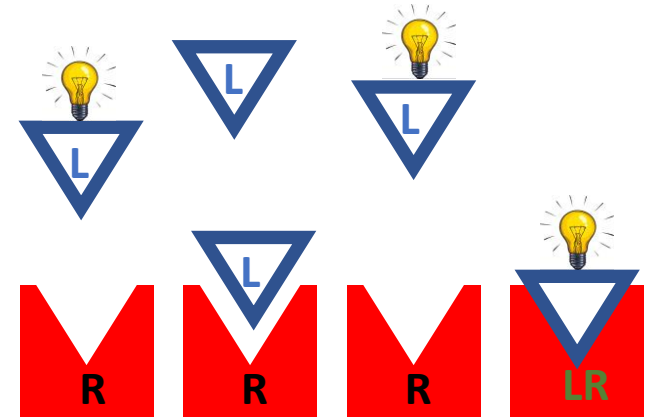
$$BP = \frac{[RL]}{[L]} = \frac{\text{"bound"}}{\text{"free"}}$$

Some useful concepts from biochemistry

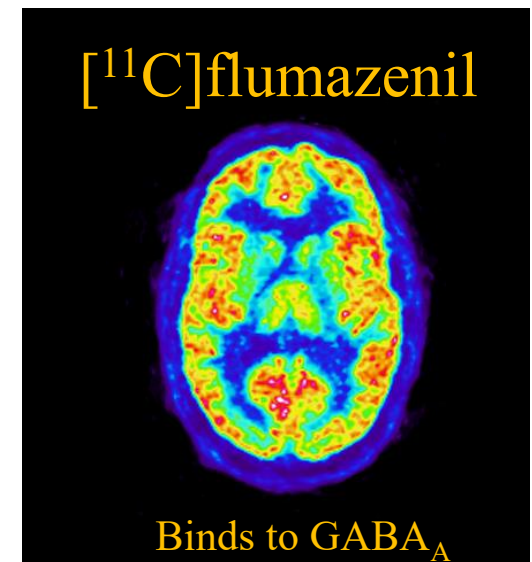
Single binding site model



$$K_D = \frac{k_{off}}{k_{on}} \quad BP = \frac{[RL]}{[L]}$$

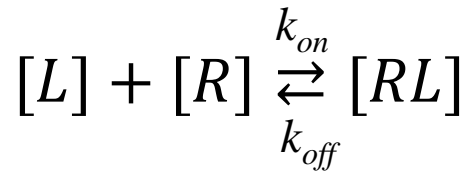


What would we ideally want from a PET experiment?

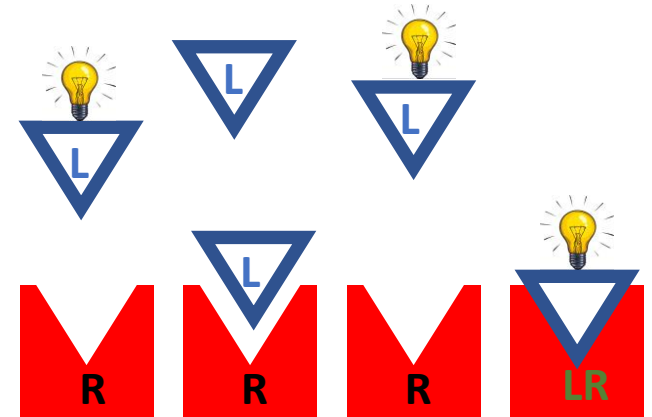


Some useful concepts from biochemistry

Single binding site model

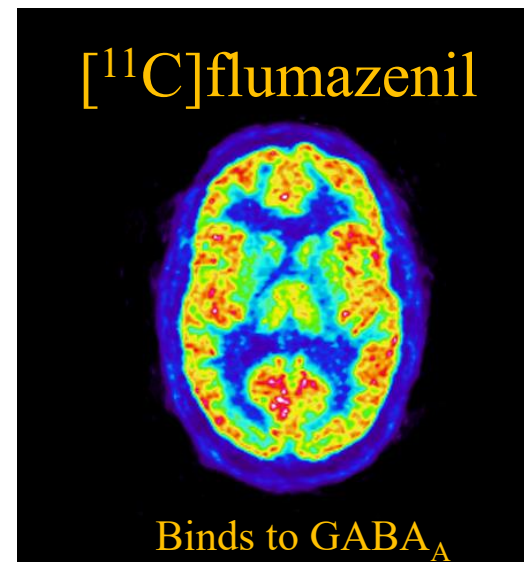


$$K_D = \frac{k_{off}}{k_{on}} \quad BP = \frac{[RL]}{[L]}$$



What would we ideally want from a PET experiment?

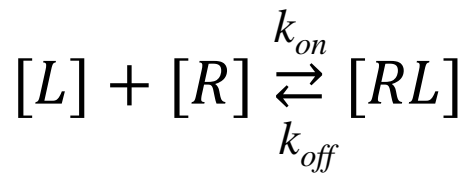
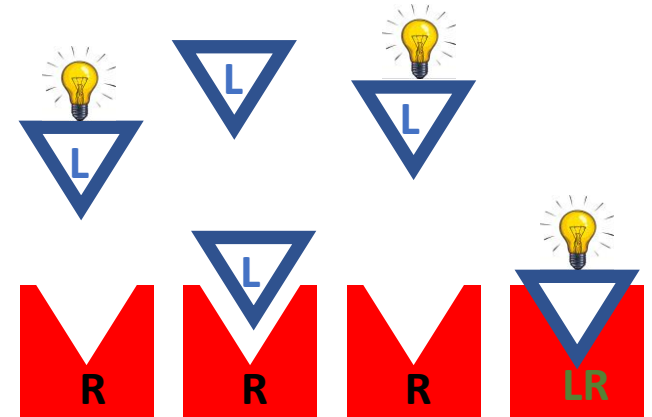
Probably we want to estimate the number of receptors, [R]
(B_{max} or B_{avail})



Some useful concepts from biochemistry

Single binding site model

We want to estimate the number of receptors, [R]
(B_{\max} or B_{avail})



$$K_D = \frac{k_{off}}{k_{on}}$$

Michelis-Menten equation

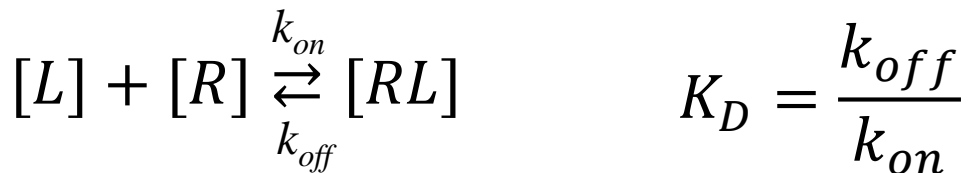
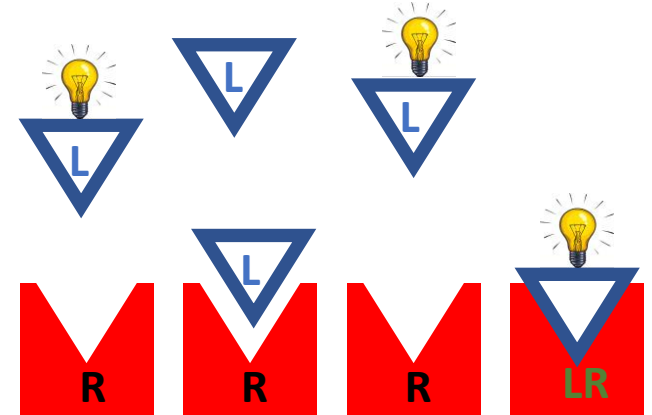
$$[RL] = \frac{[R][L]}{[L] + K_D}$$

$$BP = \frac{[RL]}{[L]}$$

Some useful concepts from biochemistry

Single binding site model

We want to estimate the number of receptors, $[R]$
(B_{\max} or B_{avail})



Michelis-Menten equation

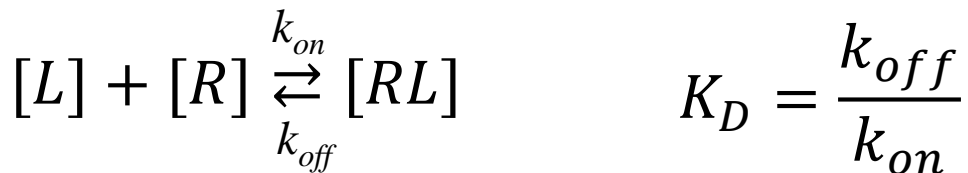
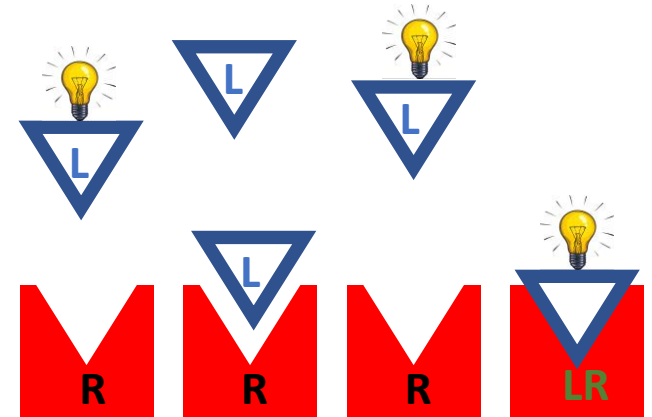
$$[RL] = \frac{[R][L]}{[L] + K_D}$$

$$BP = \frac{[RL]}{[L]} = \frac{[R][L]}{[L]([L] + K_D)}$$

Some useful concepts from biochemistry

Single binding site model

We want to estimate the number of receptors, [R]
(B_{\max} or B_{avail})



Michelis-Menten equation

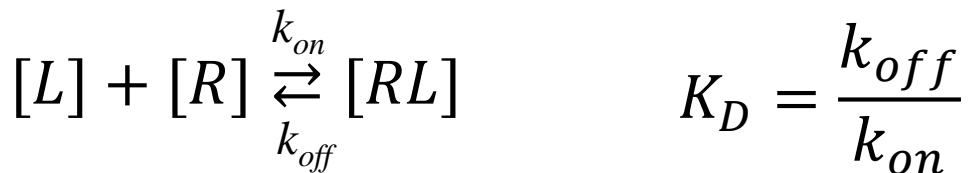
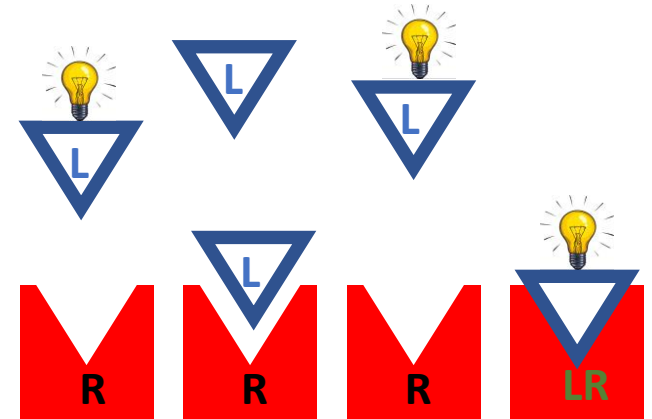
$$[RL] = \frac{[R][L]}{[L] + K_D}$$

$$BP = \frac{[RL]}{[L]} = \frac{[R][L]}{[L]([L] + K_D)} \quad \left[\text{Tracer doses: } ([L] \ll K_D) \right]$$

Some useful concepts from biochemistry

Single binding site model

We want to estimate the number of receptors, [R]
(B_{max} or B_{avail})



Michelis-Menten equation

$$[RL] = \frac{[R][L]}{[L] + K_D}$$

$$BP = \frac{[RL]}{[L]} = \frac{[R][L]}{[L]([L] + K_D)} \quad \left[\begin{array}{l} \text{Tracer doses:} \\ ([L] \ll K_D) \end{array} \right] = \frac{B_{max}}{K_D}$$

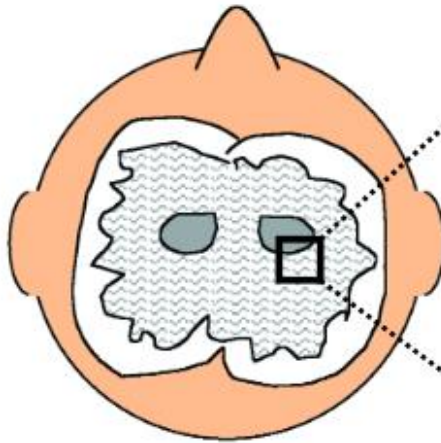
Some useful concepts from biochemistry

Single binding site model

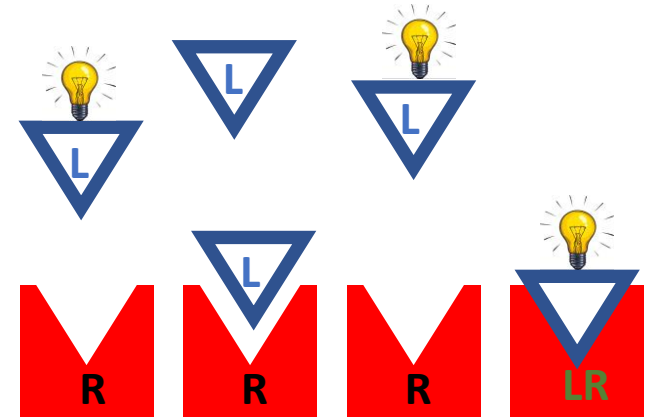
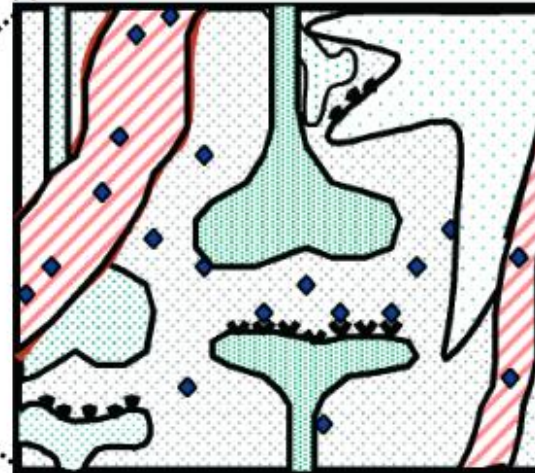
What would we ideally want from a PET experiment?

$$BP = \frac{\text{bound}}{\text{free}} = \dots = \frac{B_{max}}{K_D}$$

(a)



(b)



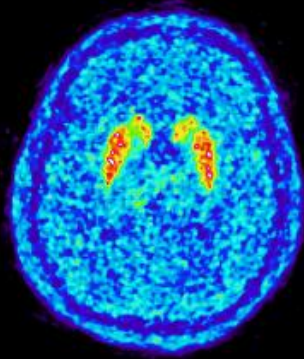
Morris et al., Emission Tomography, Elsevier 2004

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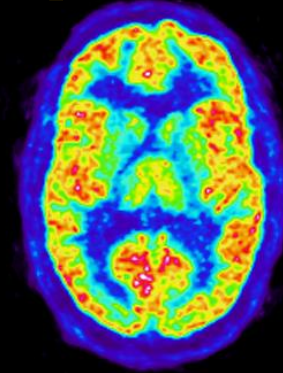
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PET/SPECT data quantification

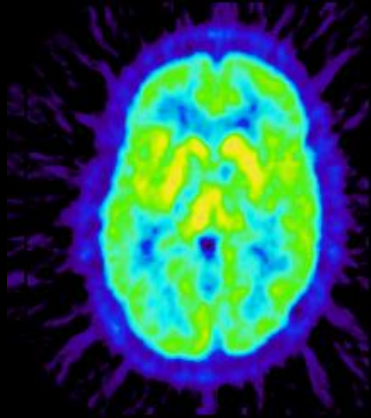
$[^{11}\text{C}]$ raclopride



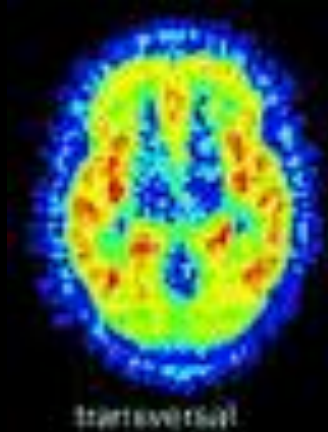
$[^{11}\text{C}]$ flumazenil



$[^{11}\text{C}]$ MADAM

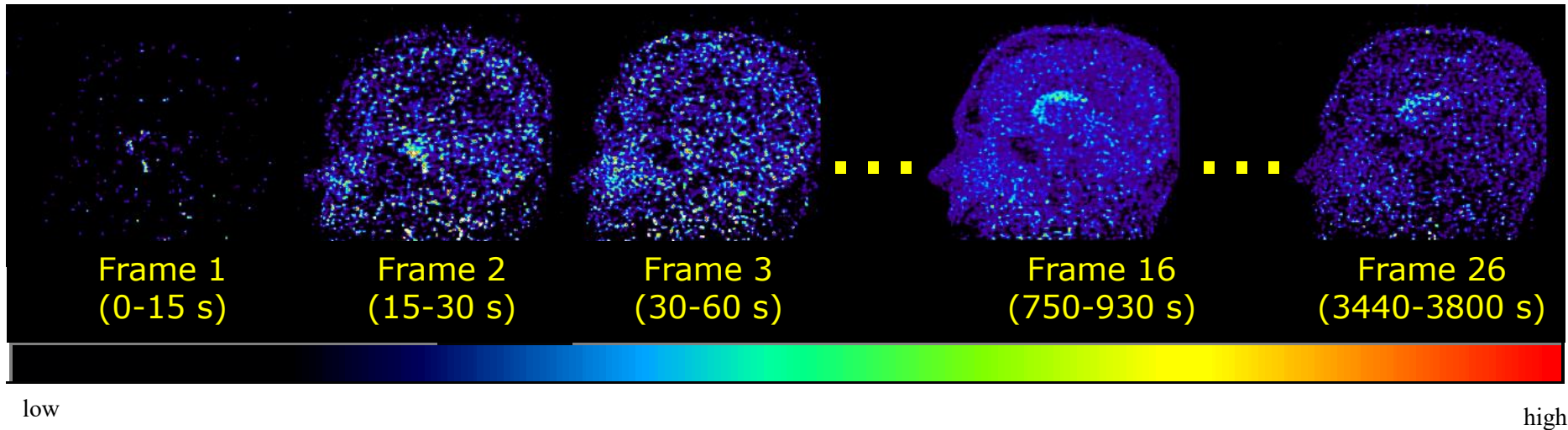


$[^{11}\text{C}]$ WAY-100635



Time frames of a PET image

time →



Increasing frame durations (why?)

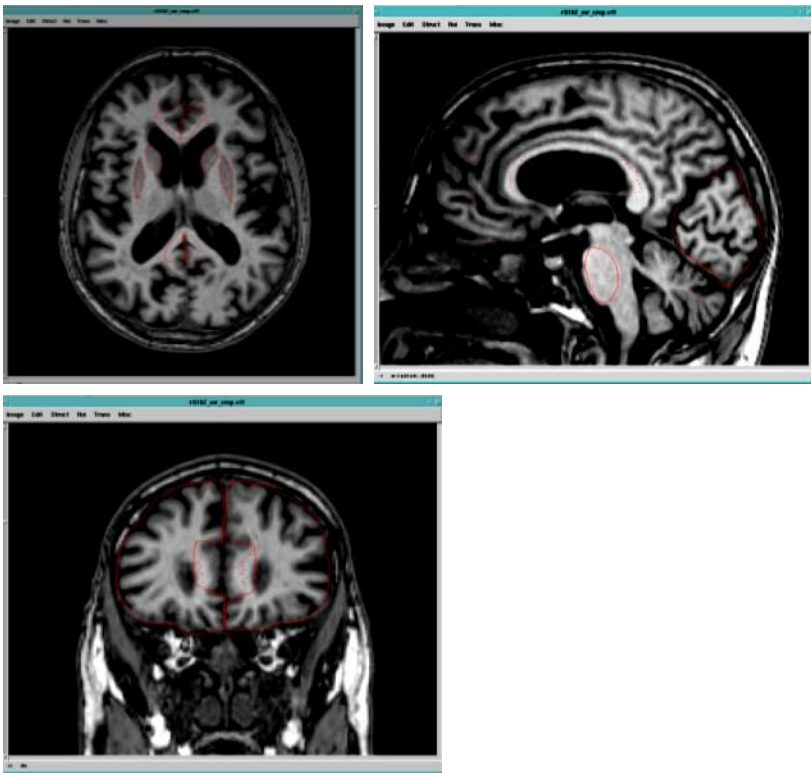
Each voxel (pixel) has a value, what's the unit?

Very noisy

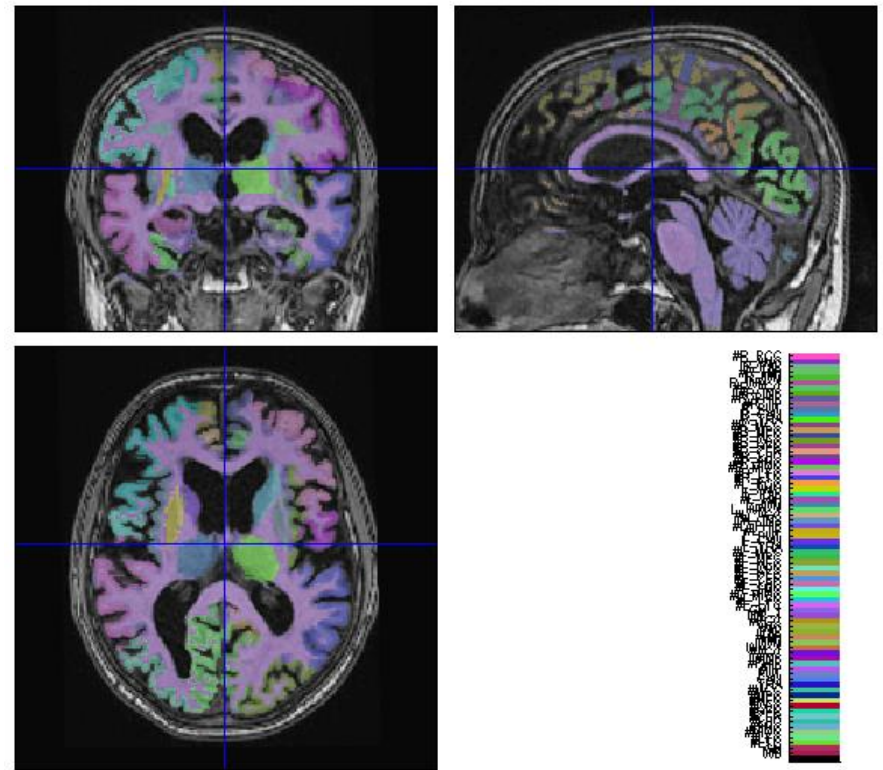
Very little spatial information

Regions of Interests (ROI)

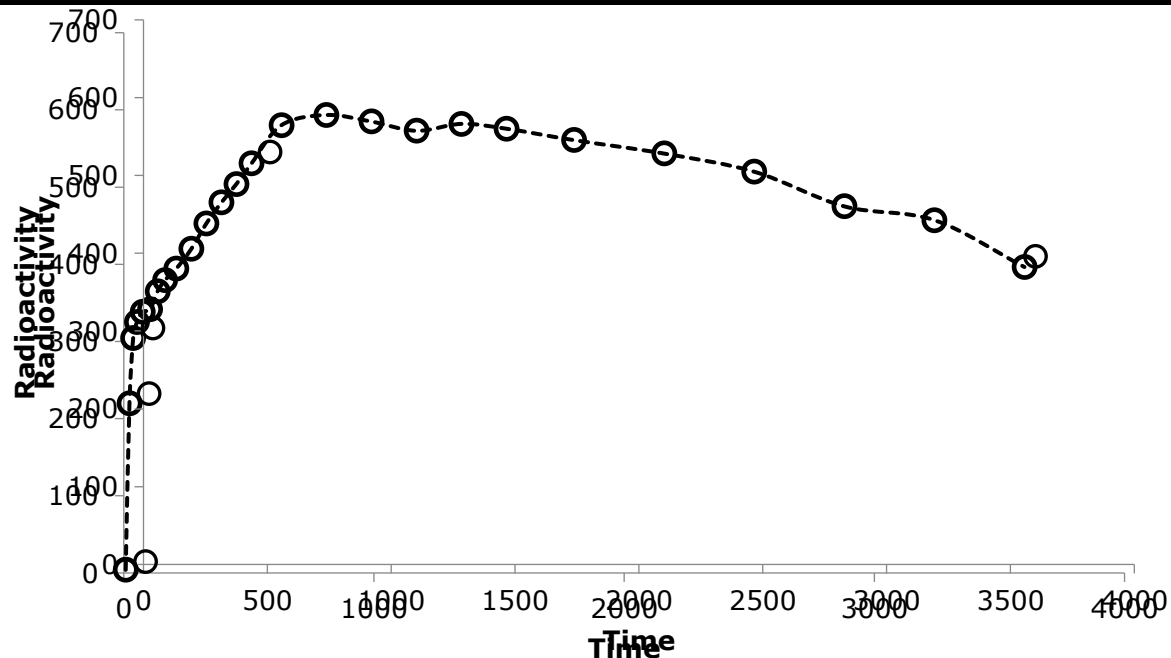
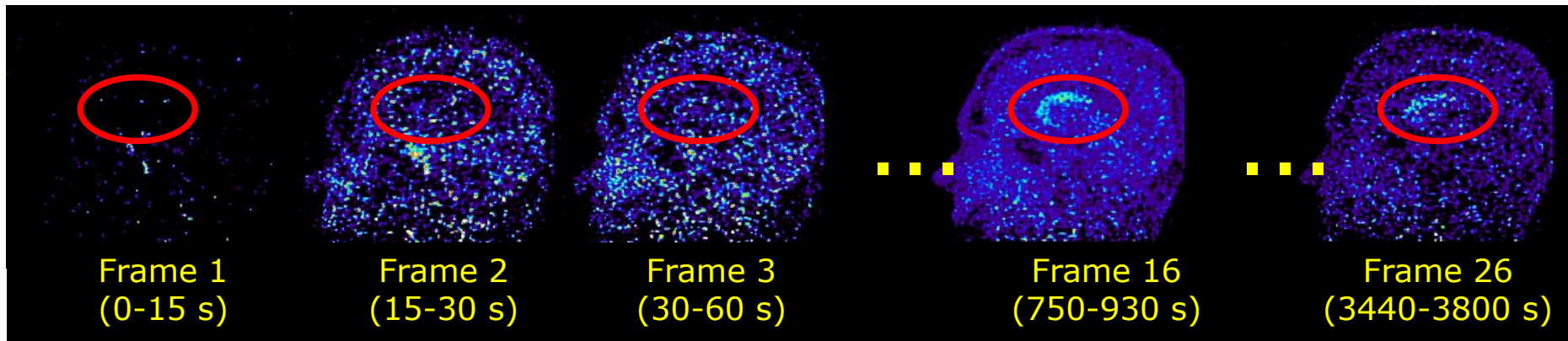
Manual ROIs



Automatic ROIs

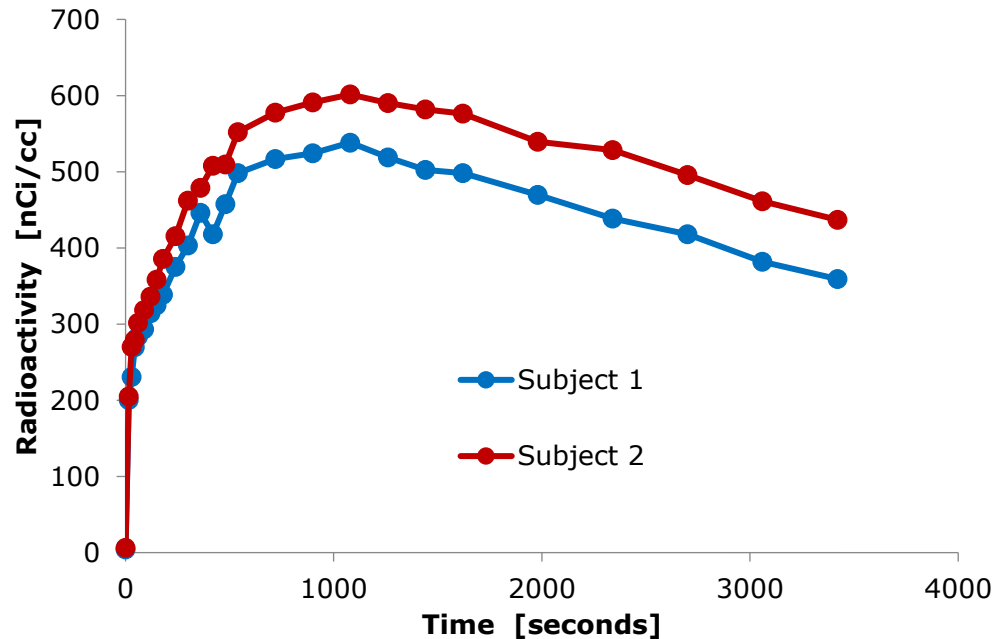


Time frames of PET / SPECT images



Time activity curve (TAC)

Quantification of dynamic PET / SPECT data



Subject 2 has a higher density of Dopamine D2 receptors !?!?

Was the same amount of radioactivity injected both subjects?

Did they have the same body weight?

Did the radioligand metabolise in the exact same way?

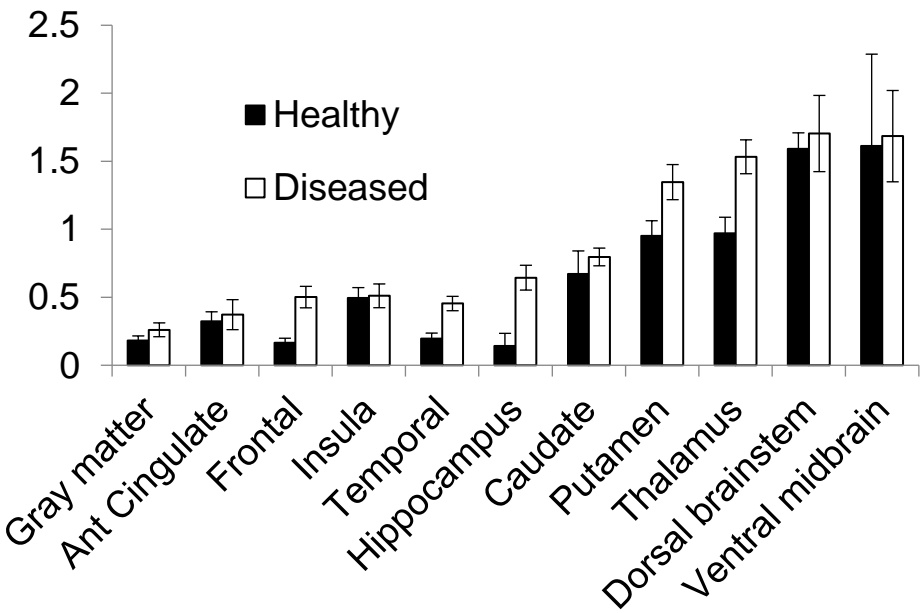
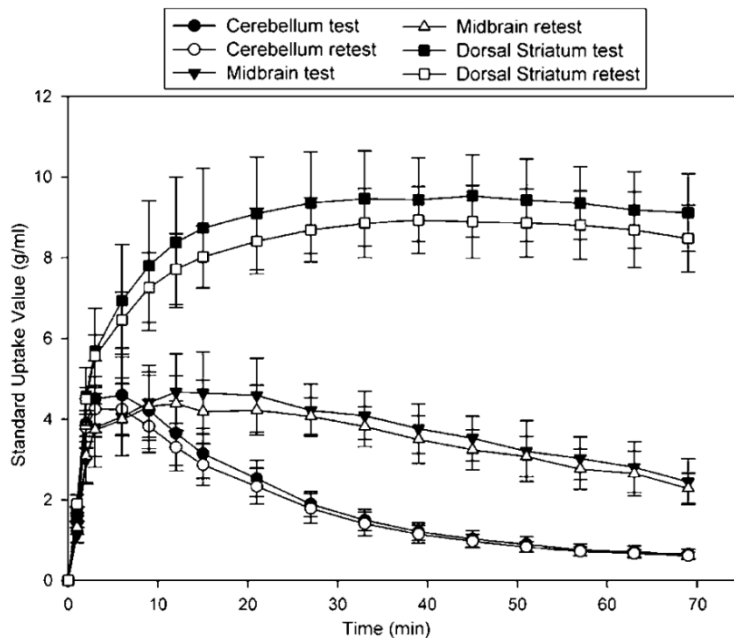
Did they have the same degree of non-specific binding?

Etc...

Standardized uptake value

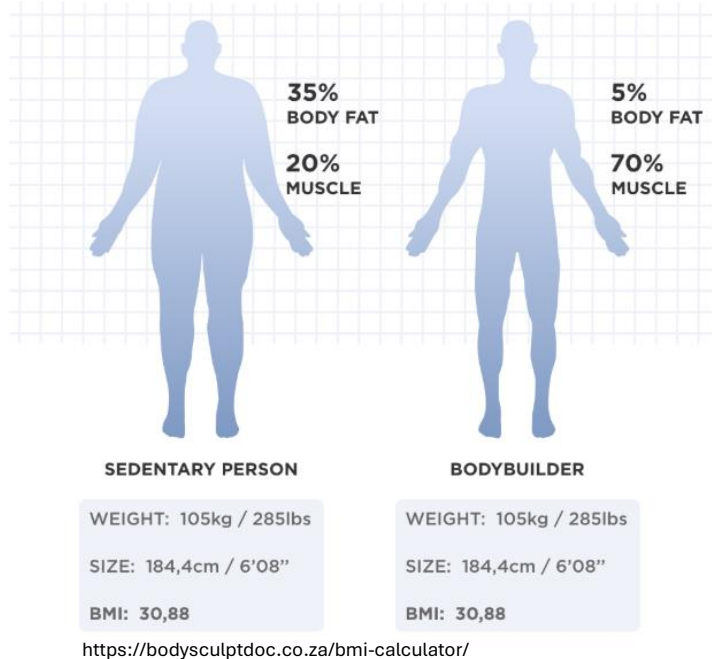
$$SUV(t) = \frac{TAC(t)}{ID / weight}$$

$$AUC_{SUV} = \int_0^{t_{end}} SUV(t) dt$$



SUV curves, [¹¹C]PE2I.
Hirvonen et al., JCBFM 2008

Example where SUV may be inappropriate



- Inject 105 MBq in each
- 5x more uptake in brain than in “background”
- Volume in which tracer is distributed
 - Sedentary: $105\text{kg} \times 0.65 = 68.25\text{kg}$
 - Bodybuilder: $105\text{kg} \times 0.95 = 99.75\text{kg}$
- Amount of “background” radioactivity per kilo lean mass
 - Sedentary: $105\text{MBq} / 68.25\text{kg} = 1.5 \text{ MBq/kg}$
 - Bodybuilder: $105\text{MBq} / 99.75\text{kg} = 1.05 \text{ MBq/kg}$
- Uptake in brain
 - Sedentary = $5 \times 1.5\text{MBq/kg} = 7.5 \text{ MBq/kg}$
 - Bodybuilder = $5 \times 1.05\text{MBq/kg} = 5.25 \text{ MBq/kg}$
- SUV:

$$SUV = \frac{\text{Uptake in brain}}{\text{injected dose} / \text{bodyweight}}$$

- Sedentary: $7.5 / 105\text{MBq} / 105\text{kg} = 7.5$
- Bodybuilder: $5.25 / 105\text{MBq} / 105\text{kg} = 5.25$

Revisiting Weight-Normalized SUV and Lean-Body-Mass-Normalized SUV in PET Studies

Ismet Sarikaya¹, Ahmed N. Albatineh², and Ali Sarikaya³

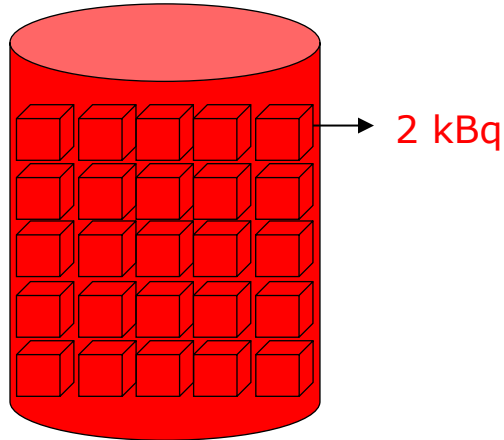
¹Department of Nuclear Medicine, Faculty of Medicine, Kuwait University, and Mubarak Al-Kabeer Hospital, Jabriya, Kuwait;

²Department of Community Medicine and Behavioral Sciences, Faculty of Medicine, Kuwait University, Safat, Kuwait; and

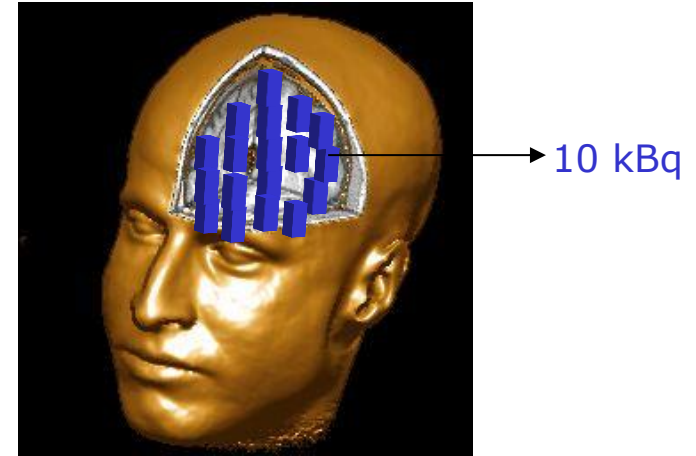
³Department of Nuclear Medicine, Faculty of Medicine, Trakya University, Edirne, Turkey

Total distribution volume, V_T

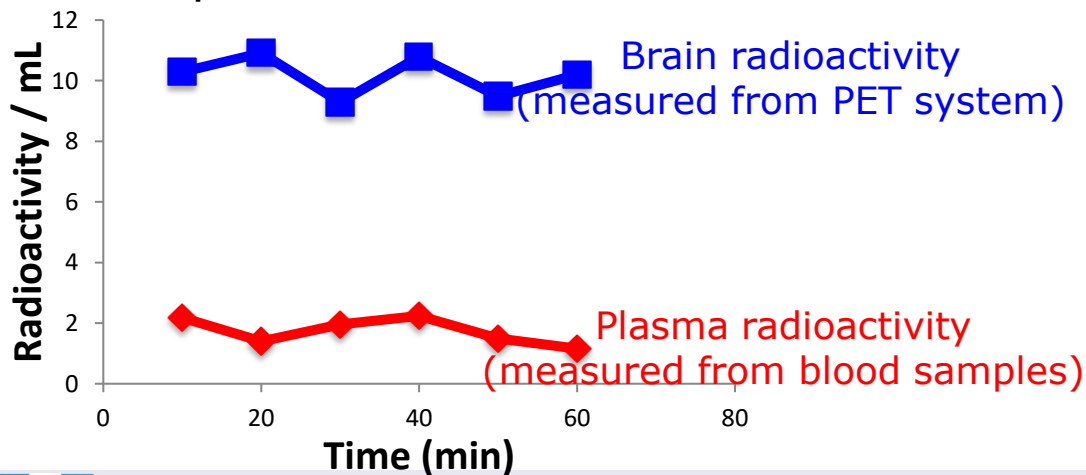
Parent radioligand in plasma



Radioactivity in brain



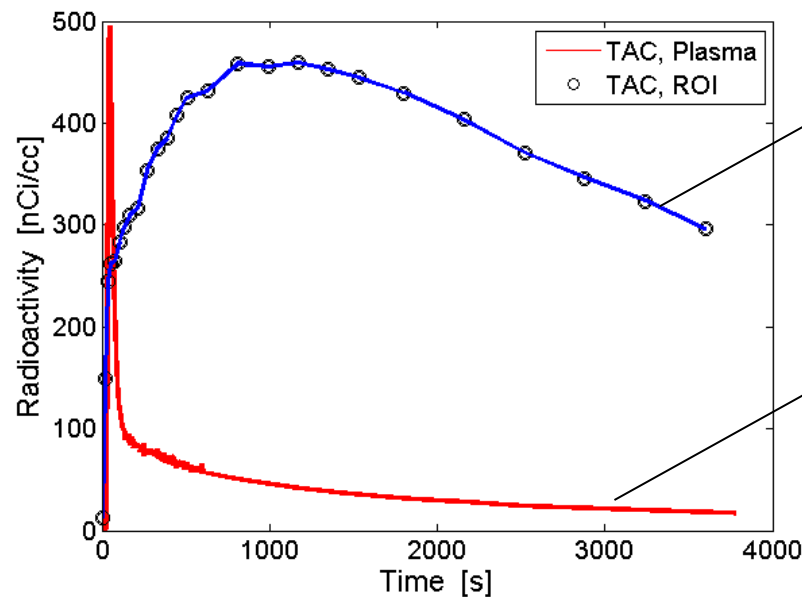
If equilibrium:



While at equilibrium,
 $V_T = \text{brain} / \text{plasma}$

Distribution volume, V_T

The volume of plasma required to account for the measured radioactivity in tissue.

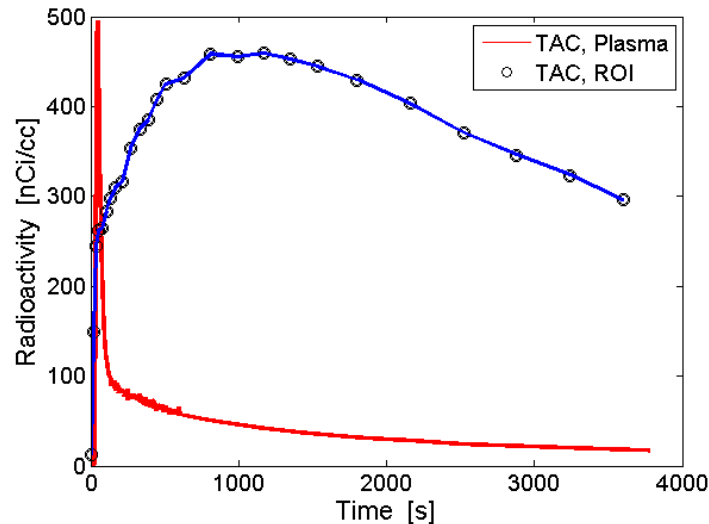


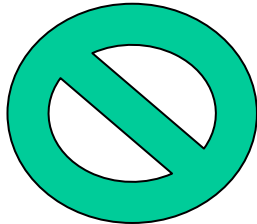
Concentration of radioligand in a brain region

Concentration of unchanged radioligand in plasma

Distribution volume, V_T

The volume of plasma required to account for the measured radioactivity in tissue.

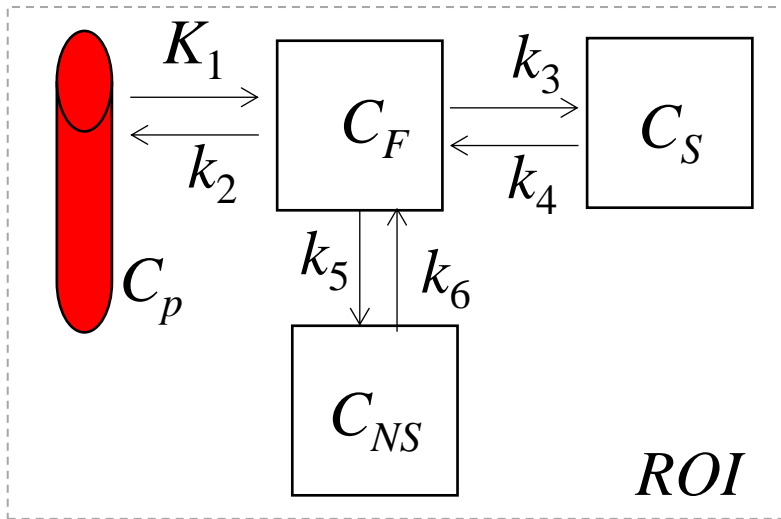


$$V_T = \frac{\int_0^{\infty} TAC(t) dt}{\int_0^{\infty} plasma(t) dt}$$


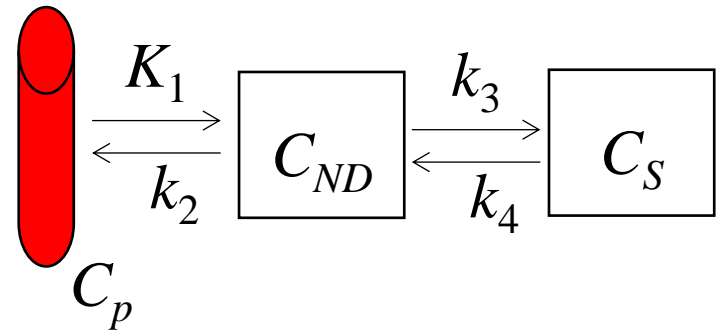
Distribution volume, V_T

What can we assume about the kinetic behaviour of the tracer?

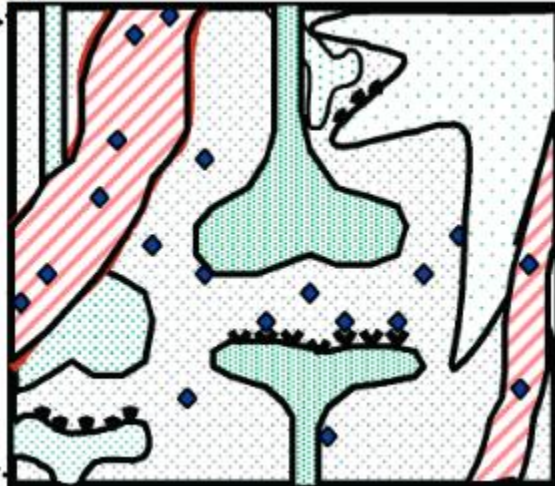
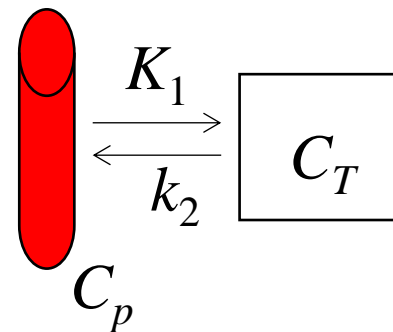
3 tissue compartment model



2 tissue compartment model

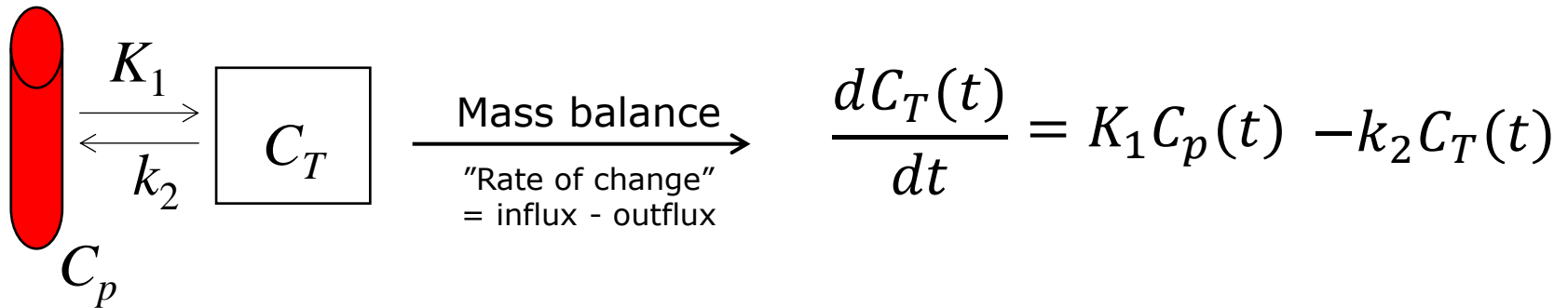


1 tissue compartment model



Distribution volume, V_T

What can we assume about the kinetic behaviour of the tracer?



Solving dif.
Eq...

$$C_T(t) = C_p(t) \otimes K_1 e^{-k_2 t}$$

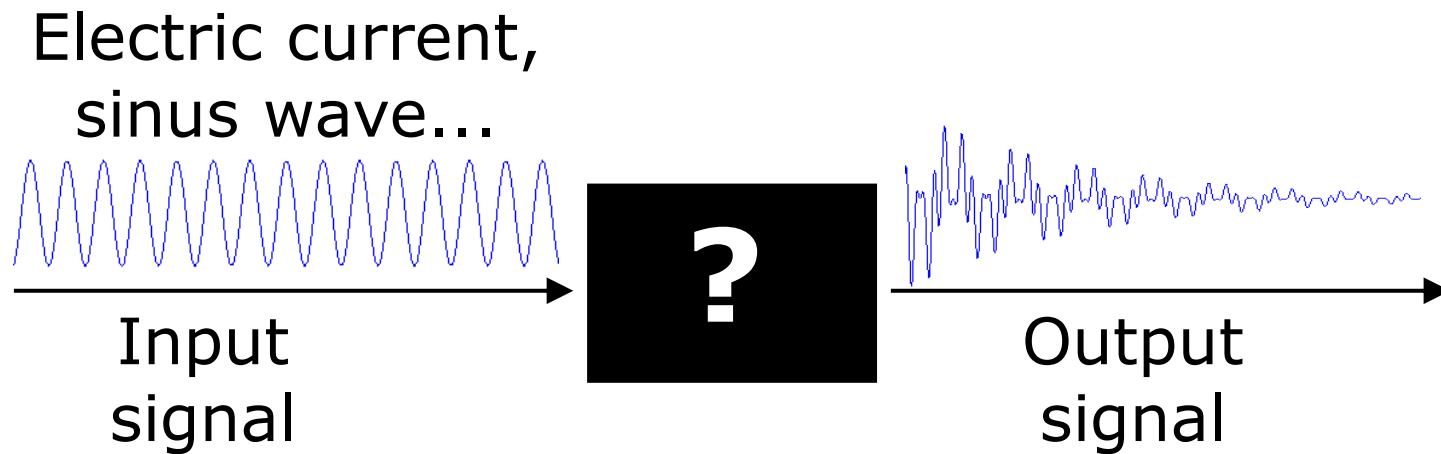
$$V_T = \frac{\int_0^{\infty} TAC(t) dt}{\int_0^{\infty} plasma(t) dt} = \frac{\int_0^{\infty} C_T(t) dt}{\int_0^{\infty} C_p(t) dt} = \dots = \frac{K_1}{k_2}$$

Linear Time Invariant (LTI) Systems



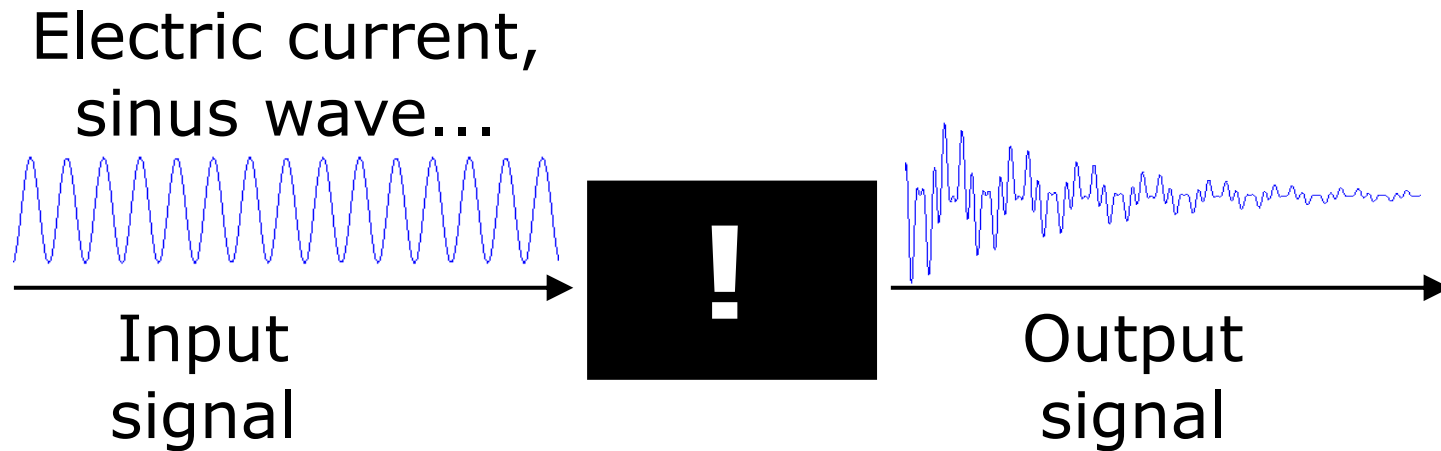
Linear Time Invariant (LTI) Systems

Synthesizer



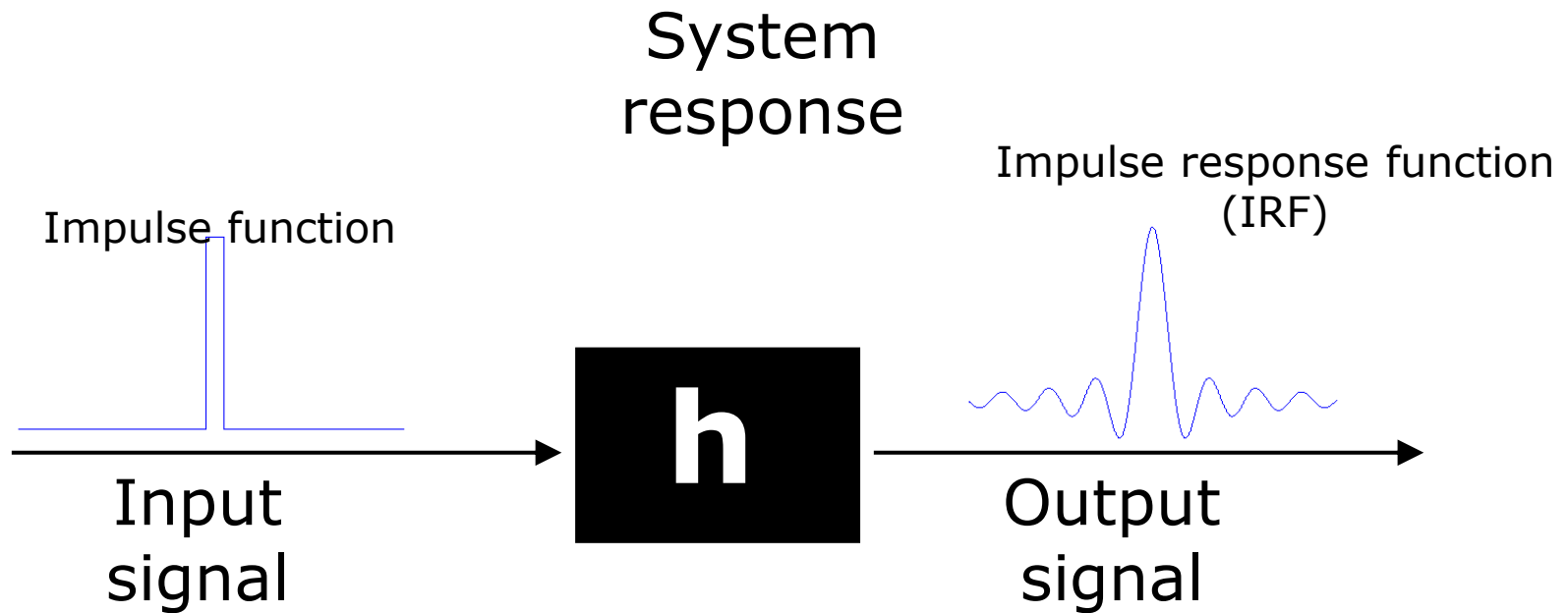
Linear Time Invariant (LTI) Systems

Synthesizer



Modify input signal + Study output signal →
Understand the system (!)

Linear Time Invariant (LTI) Systems



Impulse response function "defines" the system!

For any input signal $f_i(t)$, the corresponding output signal $f_o(t)$ is given by

$$f_o(t) = f_i(t) \otimes IRF$$

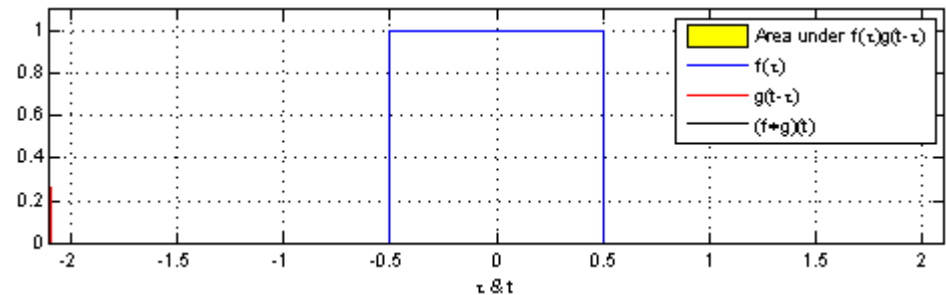
Convolution

Convolution of two square functions. The convolution (black line) at any time is the size of the joint area (yellow field) of the two functions at that time.

Convolution of a square **input function** and a **"response function"**. The convolved signal will be the output in this **"model"**.

Convolution

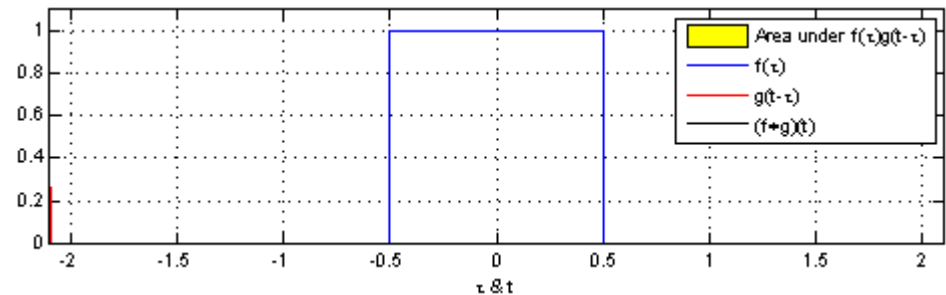
Convolution of two square functions. The convolution (black line) at any time is the size of the joint area (yellow field) of the two functions at that time.



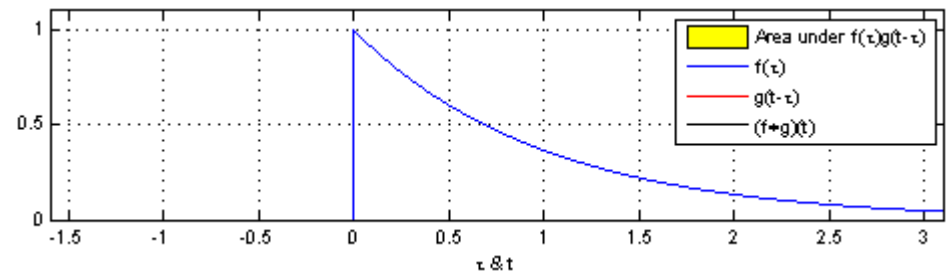
Convolution of a square **input function** and a "**response function**". The convolved signal will be the output in this "**model**".

Convolution

Convolution of two square functions. The convolution (black line) at any time is the size of the joint area (yellow field) of the two functions at that time.



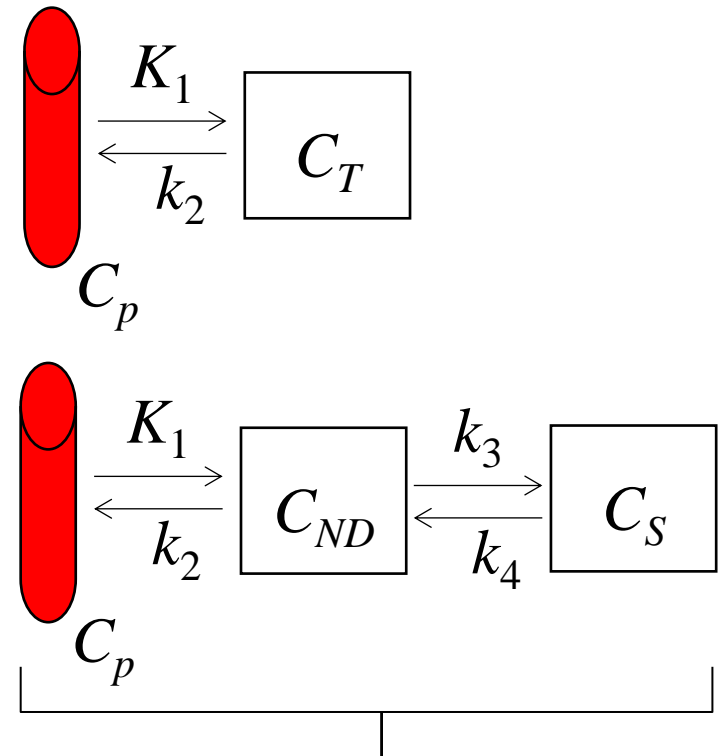
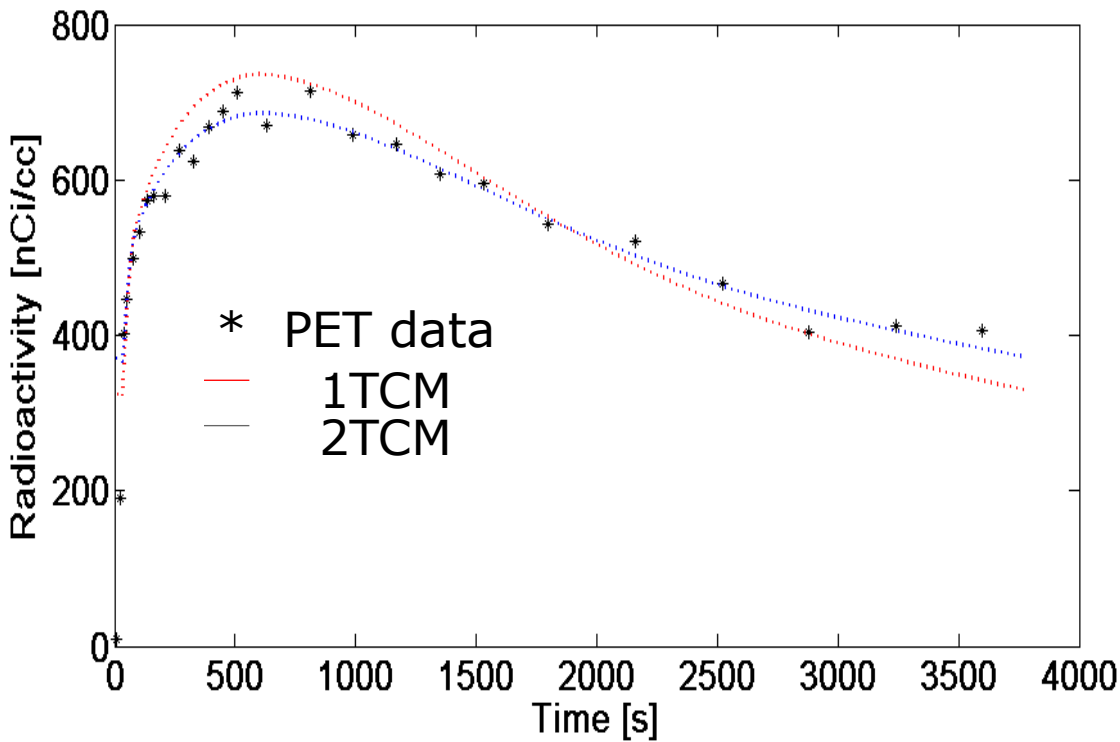
Convolution of a square **input function** and a **"response function"**. The convolved signal will be the output in this **"model"**.



Distribution volume, V_T

$$C_T(t) = C_p(t) \otimes K_1 e^{-k_2 t}$$

$$V_T = \frac{\int_0^\infty C_T(t) dt}{\int_0^\infty C_p(t) dt} = \dots = \frac{K_1}{k_2}$$



$$V_T = \frac{K_1}{k_2} \left(1 + \frac{k_3}{k_4} \right)$$

Distribution volume, V_T

Question?

What if you think that the fits obtained from 1TCM and 2TCM are equally good?

Occam's razor
Parsimony Principle

Distribution volume, V_T

Example I

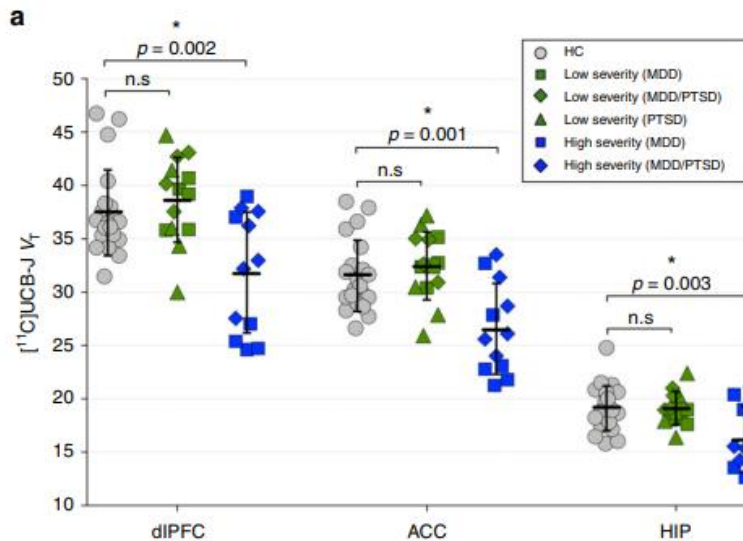
ARTICLE

<https://doi.org/10.1038/s41467-019-09562-7>

OPEN

Lower synaptic density is associated with depression severity and network alterations

Sophie E. Holmes¹, Dustin Scheinost², Sjoerd J. Finnema¹, Mika Naganawa², Margaret T. Davis¹, Nicole DellaGioia¹, Nabeel Nabulsi², David Matuskey^{1,2}, Gustavo A. Angarita¹, Robert H. Pietrzak^{1,3}, Ronald S. Duman¹, Gerard Sanacora¹, John H. Krystal^{1,3}, Richard E. Carson² & Irina Esterlis^{1,3}



Discussion

This is the first study to investigate radioligand binding to SV2A in MDD and PTSD, and the first in vivo evidence of lower synaptic density in association with depressive symptoms in these disorders. Findings suggest that lower synaptic density con-

PET image analysis. The primary outcome measure was total volume of distribution (V_T), computed parametrically using the 1 tissue (1T) compartment model and a metabolite-corrected arterial input function, as validated previously⁷⁰. Distribution volume (V_T) is the tissue-to-plasma concentration ratio at equilibrium and reflects total uptake (specific plus nonspecific binding) of the radioligand.

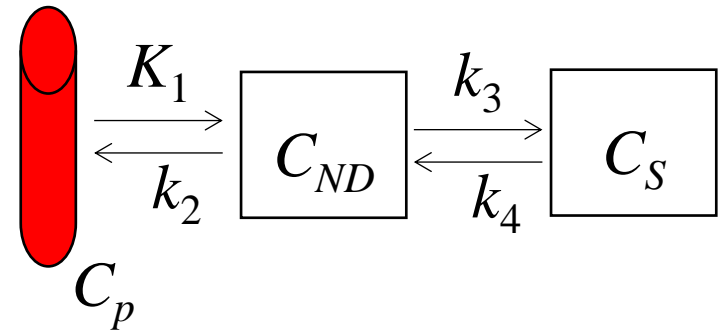
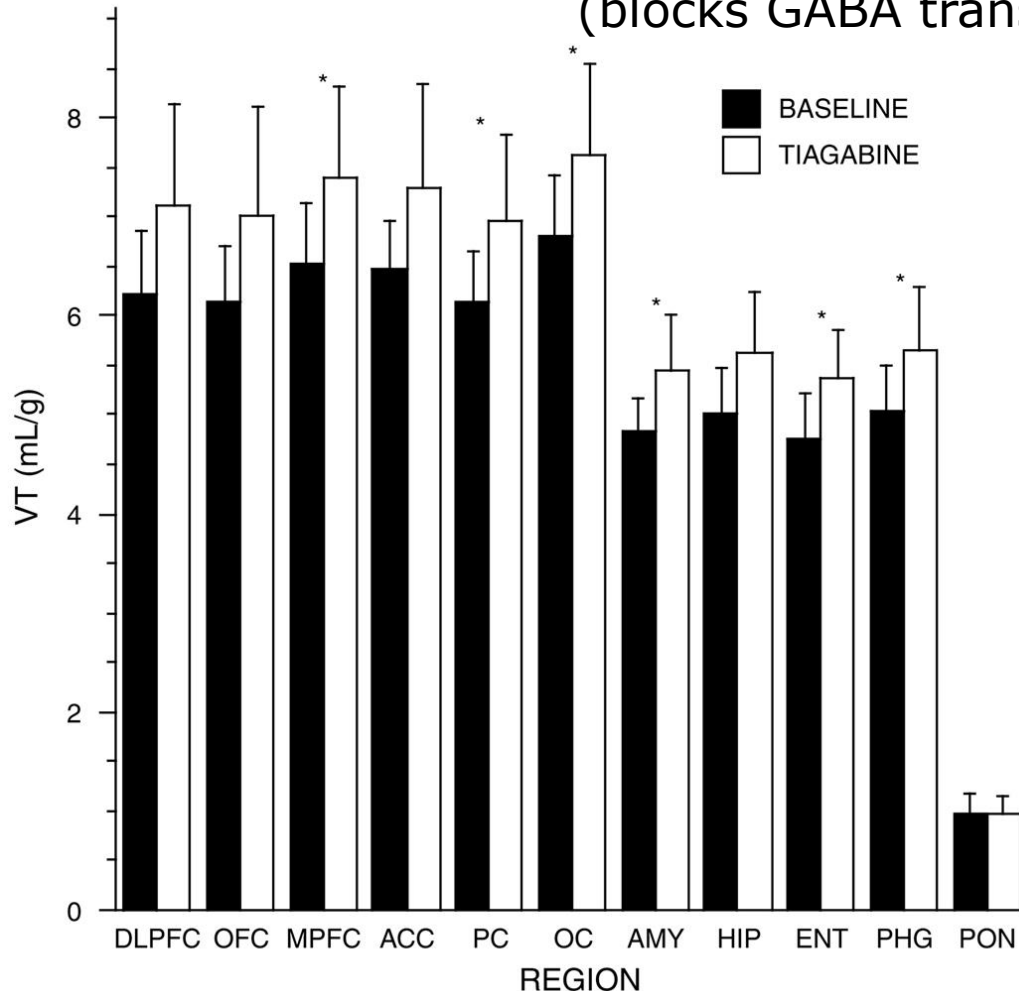
Martin Schain

NRU, Copenhagen University Hospital, Rigshospitalet

Distribution volume, V_T

Example II

[^{11}C]Flumazenil V_T before and after treatment with Tiagabine (blocks GABA transporter).



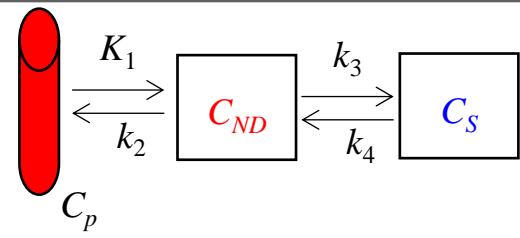
Frankle et al, *Neuropsychopharmacology* (2009)

Martin Schain

NRU, Copenhagen University Hospital, Rigshospitalet

Distribution volume, V_T

$$V_T = V_{ND} + V_S$$



Total binding
($C_T(t)$, observed)

Total binding
(modeled)

Specific binding
($C_S(t)$, modeled)

$$V_S = \frac{C_S}{C_p}$$

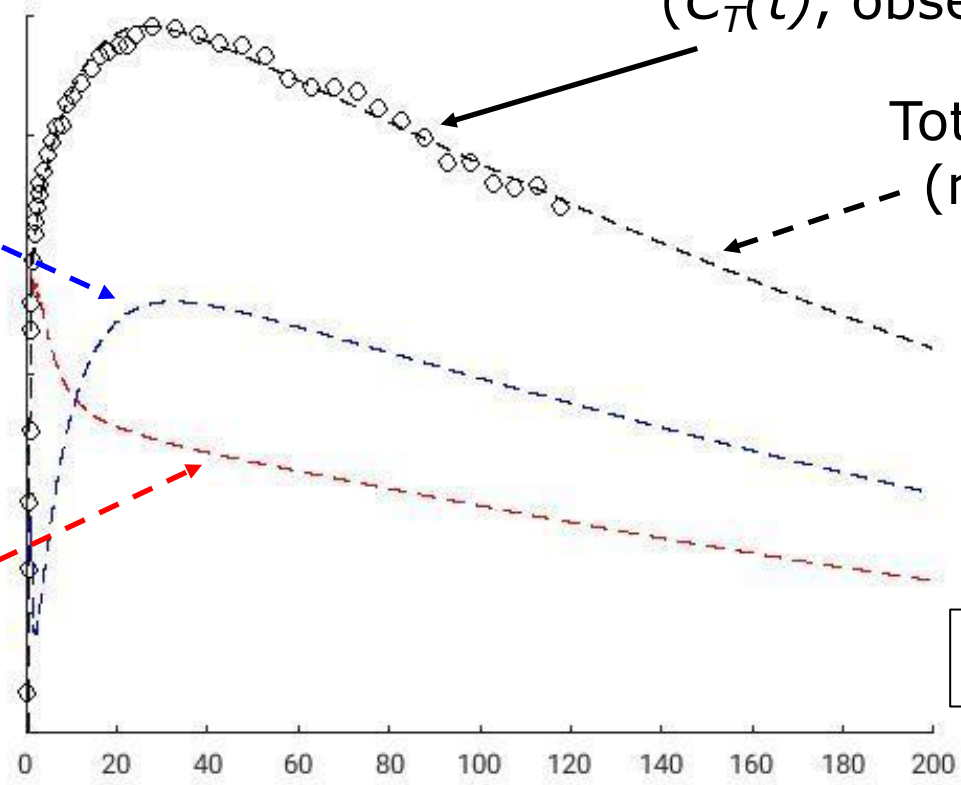
$$V_T = \frac{K_1}{k_2} \left(1 + \frac{k_3}{k_4} \right)$$

$$V_T = \frac{C_S + C_{ND}}{C_p}$$

Non-displaceable binding
($C_{ND}(t)$, modeled)

$$V_{ND} = \frac{C_{ND}}{C_p}$$

$$V_T = V_{ND} + V_S$$



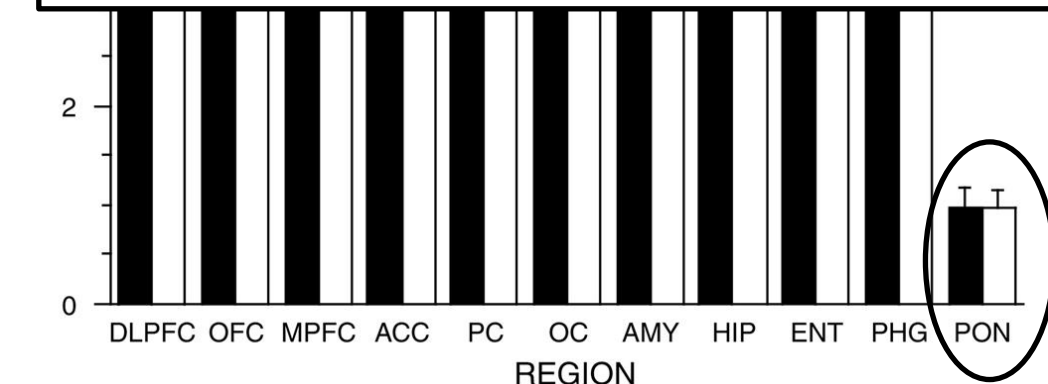
Distribution volume, V_T

Example II

[^{11}C]Flumazenil V_T before and after treatment with Tiagabine (blocks GABA transporter).

The results of this study are consistent with the hypothesis that the acute increases in extracellular cortical GABA can be detected as an increase in the binding of the BDZ site-specific radiotracer, [^{11}C]flumazenil. The principle underlying this hypothesis is the 'GABA shift'—the enhancement in BDZ-receptor affinity for BDZ site substrates resulting from the increased GABA (Tallman *et al*, 1978; Braestrup *et al*, 1982). It is widely accepted that

Derivation of BDZ parameters was based upon the following assumptions: (1) because of the low density of BDZ in the pons (Abadie *et al*, 1992; Price *et al*, 1993), pons V_T was assumed to be representative of equilibrium nonspecific binding, V_{ND} ; (2) the nonspecific binding did not vary significantly between regions.



C_p

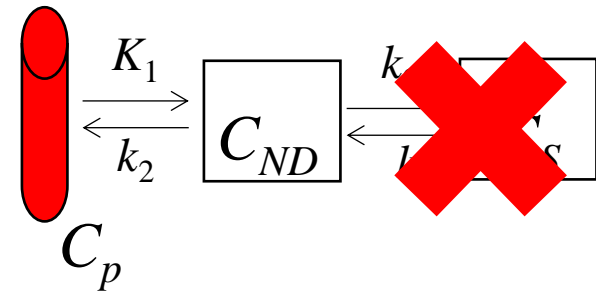
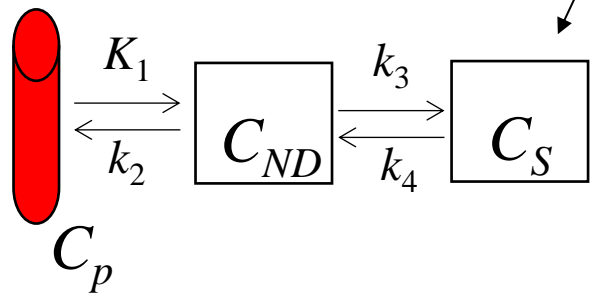
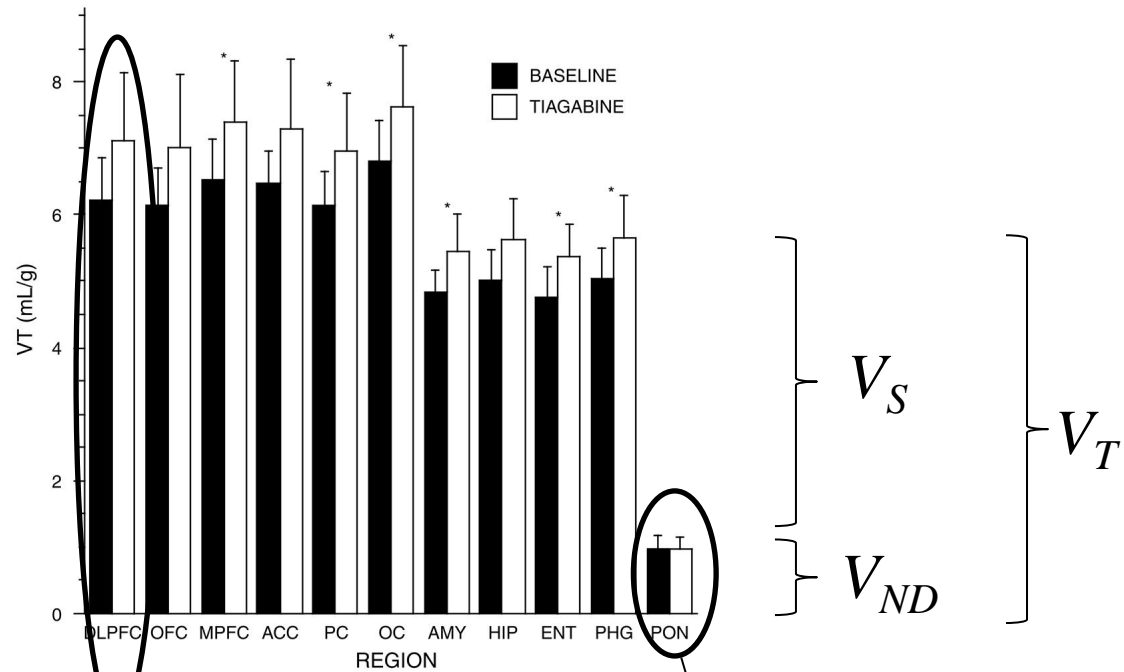
Reference region – negligible density of target
 $C_S=0$

Frankle *et al*, *Neuropsychopharmacology* (2009)

Fundamental assumption of PET:

V_{ND} doesn't change

$$V_T = V_S + V_{ND}$$



How can we estimate V_{ND} ?

If there is a reference region:

Your answer here...

If there is not a reference region:

Your answer here...

How can we estimate V_{ND} ?

If there is a reference region:

Apply your kinetic model to the reference region TAC.
The V_T you get will be V_{ND}

If there is not a reference region:

Your answer here...

How can we estimate V_{ND} ?

If there is a reference region:

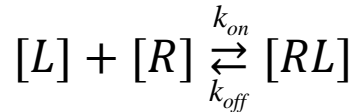
Apply your kinetic model to the reference region TAC.
The V_T you get will be V_{ND}

If there is not a reference region:

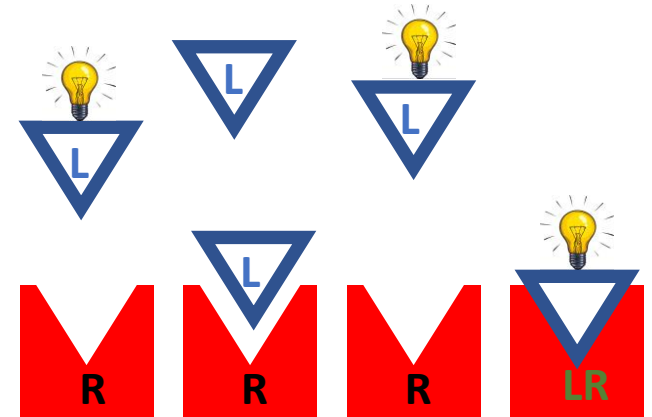
No established way exists...

Some useful concepts from biochemistry

Single binding site model (equilibrium)



$$K_D = \frac{k_{off}}{k_{on}} \quad BP = \frac{[RL]}{[L]} = \frac{B_{max}}{K_D}$$

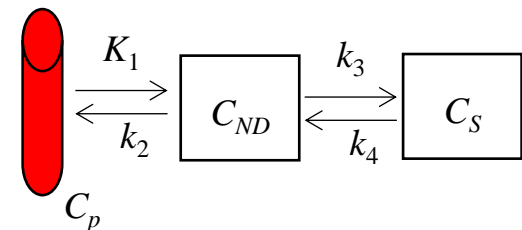


f_{ND} : fraction of free tracer in ND compartment
 f_p : fraction of free tracer in plasma

$$[RL]: C_S$$

$$[L]: f_{ND} C_{ND}$$

$$[L]: f_p C_p$$



Three approaches to estimate in vitro BP

Approach 1a (relative to free conc in plasma)

$$BP = \frac{[RL]}{[L]} = \frac{B_{max}}{K_D}$$

$$\begin{aligned} [RL]: C_S \\ [L]: f_{ND} C_{ND} \\ [L]: f_p C_p \end{aligned}$$

f_{ND} : fraction of free tracer in ND compartment
 f_p : fraction of free tracer in plasma
 C_p : Concentration of tracer in plasma

Approach 1a: "Free" means "free in plasma"

(i.e., conc. of free in plasma = conc. of free in tissue)

$$BP = \frac{[RL]}{[L]} = \frac{C_S}{f_p C_p} = \frac{C_S / C_p}{f_p C_p / C_p} = \frac{V_S}{f_p} = \frac{V_T - V_{ND}}{f_p} = \frac{B_{max}}{K_D} = BP_F$$

Three approaches to estimate in vitro BP

Approach 1b (relative to concentration in plasma)

$$BP = \frac{[RL]}{[L]} = \frac{B_{max}}{K_D}$$

$$\begin{aligned} [RL]: C_S \\ [L]: f_{ND} C_{ND} \\ [L]: f_p C_p \end{aligned}$$

f_{ND} : fraction of free tracer in ND compartment
 f_p : fraction of free tracer in plasma
 C_p : Concentration of tracer in plasma

Approach 1b: "Free" means "total in plasma"

(Conc. of free in tissue = conc. of total in plasma – e.g., f_p doesn't change across groups)

$$BP = \frac{[RL]}{[L]} = \frac{C_S}{C_p} = V_S = V_T - V_{ND} = f_p \frac{B_{max}}{K_D} = BP_P$$

Three approaches to estimate BP

Approach 2 (relative to ND)

$$BP = \frac{[RL]}{[L]} = \frac{B_{max}}{K_D}$$

$$\begin{aligned} [RL]: C_S \\ [L]: f_{ND} C_{ND} \\ [L]: f_p C_p \end{aligned}$$

f_{ND} : fraction of free tracer in ND compartment
 f_p : fraction of free tracer in plasma
 C_p : Concentration of tracer in plasma

Approach 2: "Free" means "free in non-displaceable compartment"

$$\frac{[RL]}{[L]} = \frac{C_S}{f_{ND} C_{ND}} = \frac{C_S / C_p}{f_{ND} C_{ND} / C_p} = \frac{V_S}{f_{ND} V_{ND}} \rightarrow \frac{V_T - V_{ND}}{V_{ND}} = f_{ND} \frac{B_{max}}{K_D} = BP_{ND}$$

Binding potential!

PET

B_{max} : Total number of receptors

K_D : Affinity of the radioligand

f_p : Free fraction of radioligand in plasma

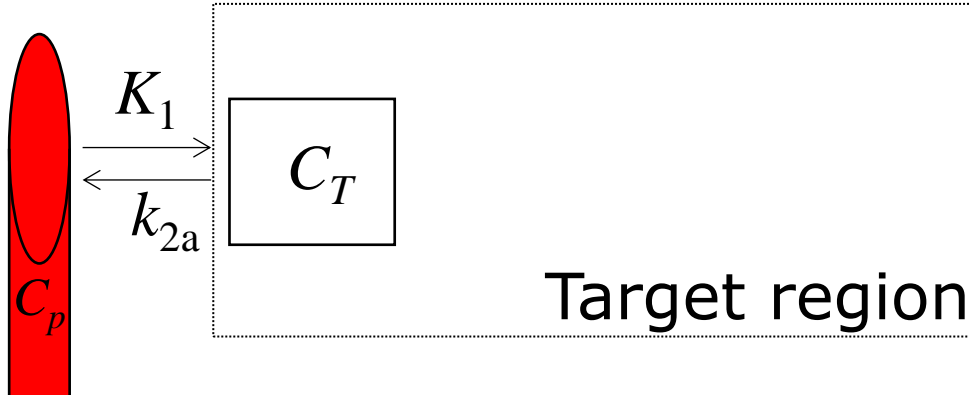
f_{ND} : Free fraction of radioligand in non-displaceable compartment

$$BP = \frac{B_{max}}{K_D}$$

Three arrows branch from the right side of the equation above to the following three equations:

$$\frac{V_T - V_{ND}}{f_p} = \frac{B_{max}}{K_D} = BP_F$$
$$V_T - V_{ND} = \frac{B_{max}}{K_D} f_p = BP_P$$
$$\frac{V_T - V_{ND}}{V_{ND}} = \frac{B_{max}}{K_D} f_{ND} = BP_{ND}$$

Simplified Reference Tissue Model



$$C_T(t) = C_p(t) \otimes K_1 e^{-k_2 t}$$

Summary of (some) assumptions

If you use 2TCM to estimate V_T :

- Assume that the model describes the data well
- Assume that V_{ND} is not different between the groups

If you use 2TCM to estimate $BP_{ND} (V_T - V_{ND}) / V_{ND}$:

- Assume that the model describes the data well
- Assume that V_{ND} is the same in all brain regions
- Assume that V_{ND} is not different between the groups

If you use reference tissue modeling to estimate BP_{ND}

- Assume that the model describes the data well
- Assume that V_{ND} is the same in all brain regions
- Assume that V_{ND} is not different between the groups

Summary of (some) assumptions

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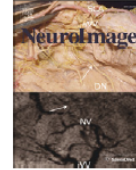
Possible to estimate V_{ND} without reference region?



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Estimation of in vivo nonspecific binding in positron emission tomography studies without requiring a reference region



R. Todd Ogden^{a,b,*}, Francesca Zanderigo^b, Ramin V. Parsey^c

^a Department of Biostatistics, Columbia University, 10032, New York, NY, USA

^b Department of Molecular Imaging and Neuropathology, New York State Psychiatric Institute, 10032, New York, NY, USA

^c Departments of Psychiatry and Radiology, Stony Brook University, Stony Brook, 11794, NY, USA



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Accuracy and reliability of [¹¹C]PBR28 specific binding estimated without the use of a reference region

Pontus Plaven-Sigray^{a,*}, Martin Schain^b, Francesca Zanderigo^{b,c}, Karolinska [¹¹C]PBR28 study group Ilan Rabiner^d, Roger Gunn^{d,e}, Todd Ogden^{b,c,f}, Simon Cervenka^a

^a Department of Clinical Neuroscience, Center for Psychiatry Research, Karolinska Institutet and Stockholm County Council, SE-171 76, Stockholm, Sweden

^b Department of Psychiatry, Columbia University, New York, NY, USA

^c Molecular Imaging and Neuropathology Division, New York State Psychiatric Institute, New York, USA

^d Invicro LLC, London, UK

^e Division of Brain Sciences, Imperial College London, London, UK

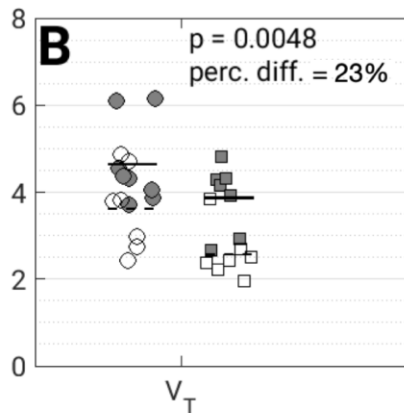
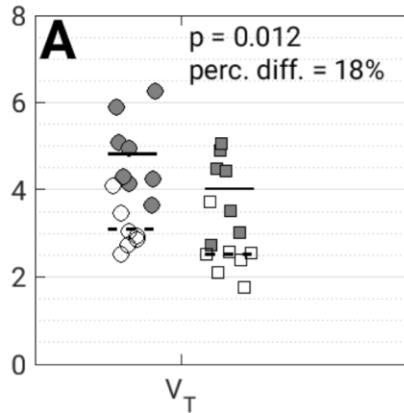
^f Department of Biostatistics, Mailman School of Public Health, Columbia University, New York, USA

Martin Schain

NRU, Copenhagen University Hospital, Rigshospitalet



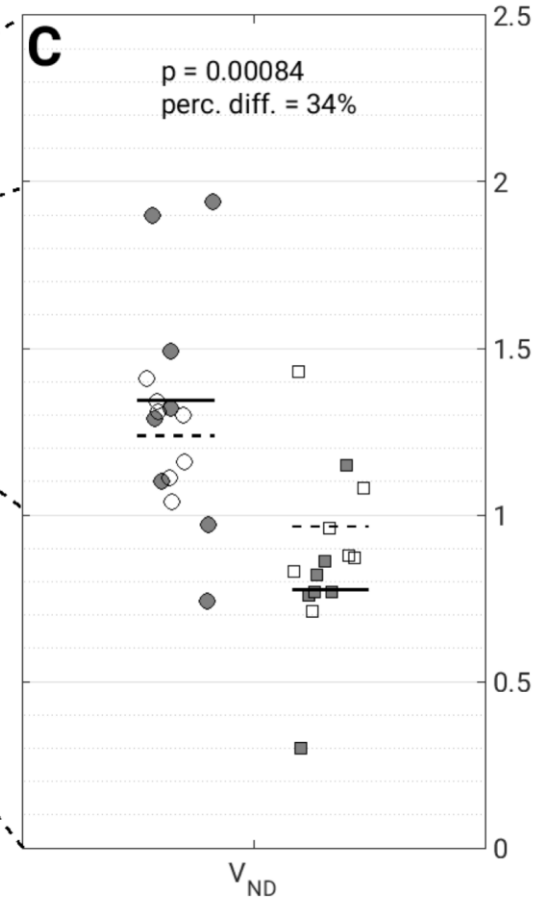
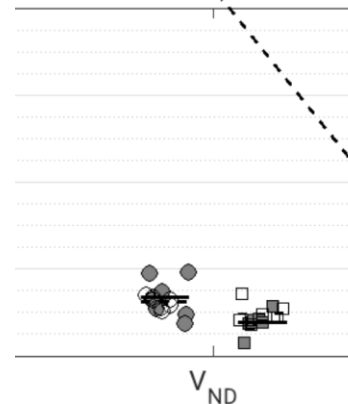
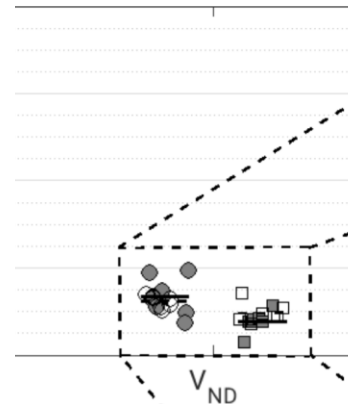
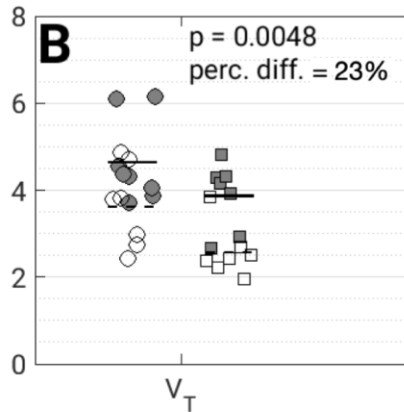
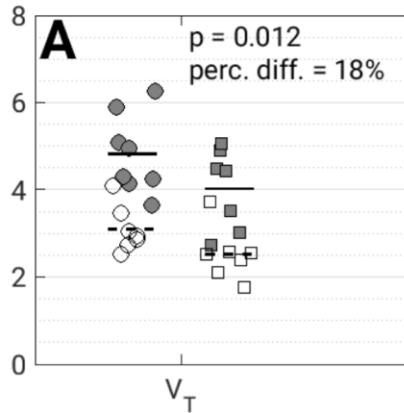
Example where V_{ND} may confound



● Ctrl (HAB) ○ Ctrl (MAB) ■ AUD (HAB) □ AUD (MAB) — mean (HAB) - - - mean (MAB)

Laurell et al., in Review

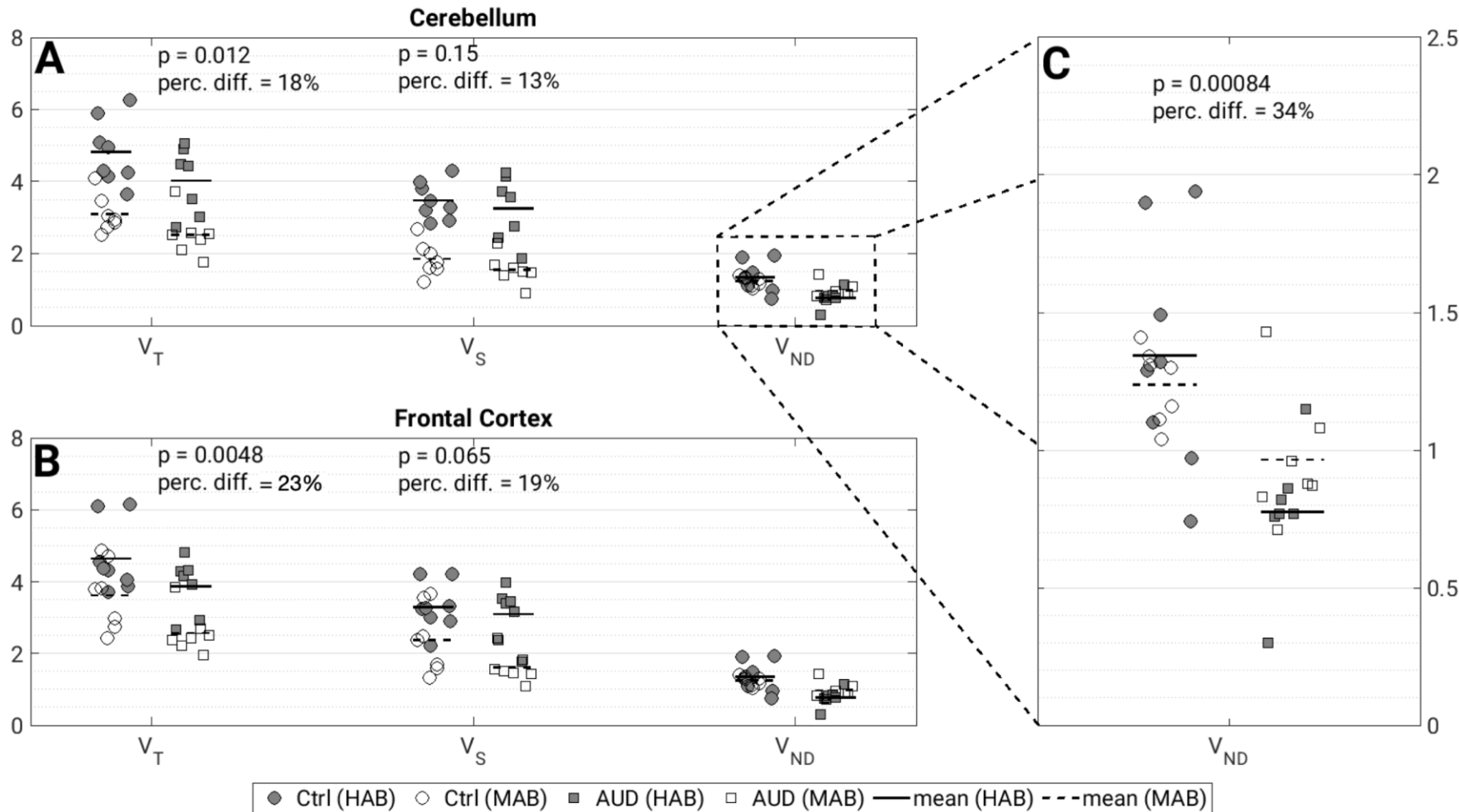
Example where V_{ND} may confound



● Ctrl (HAB) ○ Ctrl (MAB) ■ AUD (HAB) □ AUD (MAB) — mean (HAB) - - - mean (MAB)

Laurell et al., in Review

Example where V_{ND} may confound



Laurell et al., in Review

Summary

- Dynamic PET data = Acquired over time → time activity curves
- Most (not all) radioligands can be described by 1TCM or 2TCM
- 1TCM and 2TCM requires arterial input functions (cumbersome measurement)
- With a TCM, non-linear regression is used to estimate rate constants, which are combined into total distribution volume (V_T)
- If a reference region exist, non-displaceable distribution volume (V_{ND}) can be estimated → estimation of BP
- BP_{ND} , BP_F and BP_p are thought to represent estimates of B_{max} . This relies on a number of assumptions.